Velocities in Special Relativity are not Vectors

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In vector calculus the addition of two vectors means no more than the addition of their components. According to Special Relativity Theory (SRT) the addition of two velocities signifies more than that. Apart from adding the components we have to divide their sum by a coefficient 1+(**v**/c)(**u**/c), which presence indicates that the velocity is not a vector in SRT. If the velocities in SRT were vectors they should be added up as vectors. The ratio of velocities **v**/c should not be considered as the hyperbolic tangent of an angel. The coefficient mentioned above appeared as the result of the baseless introduction of hyperbolic functions to the SRT formulae. On that basis the formulae although fully consistent are evidently wrong. We have shown that they can easily be reduced to the correct formulae of the Galilean Transform. Also we have shown that in the case of a 3D space there are three different coefficients and three corresponding different times for one moving object. Therefore there is a choice to be made. On the one hand by dismissing the hyperbolic functions from SRT we annihilate SRT and on the other, by accepting them we reject (commonly accepted) the rules of vector calculus and obtain in 3D case, three different times instead of one.

1. Introduction

The rules of vector calculus are not complicated. An algebraic sum of two vectors **v** and **u** means addition of their components in such a way that the resulting vector in 3D space is given by:

$$\mathbf{v}' = \mathbf{v} - \mathbf{u} = (v_x - u_x)\mathbf{i} + (v_y - u_y)\mathbf{j} + (v_z - u_z)\mathbf{k} \tag{1}$$

where $\mathbf{i},\mathbf{j},\mathbf{k}$ are unit vectors.

Of course in 1D space the indexes are useless and the equation (1) simply reduces to

$$v' = v - u \tag{2}$$

In SRT, Eq. (2) is somewhat different. According to Einstein, in the case of the object velocity **v** and observer velocity **u**, the following equation should be used:

$$\mathbf{v}' = \frac{\mathbf{v} - \mathbf{u}}{1 - vu/c^2} \quad \Rightarrow \quad \frac{\mathbf{v}'}{c} = \frac{\mathbf{v}/c - \mathbf{u}/c}{1 - \frac{v}{c}\cdot\frac{u}{c}} \tag{3}$$

Can we guess why and how Einstein changed the rules of summing up velocities? The answer for the former question is simple: to secure his statement that there is no velocity higher than velocity of light and to make his formula for velocity, consistent with that of other formulae of SRT.

The answer for the latter question is unbelievable. It seems that Einstein made a silent assumption that ratio of any velocity to phase velocity of light, equals a hyperbolic tangent of an argument called rapidity [4].

We have to admit that Einstein formula for velocities addition is fully consistent with other formulae of SRT. We have already shown that in the case of Minkowski space-time [1]. Yet another possible way of obtaining Einstein's formulae for velocity addition is presented in the next chapter.

2. Einstein Velocity from a Lorentz Transform

The Einstein Eq. (3) for velocity addition can also be obtained starting from the Lorentz transform Eqs. (4-5):

$$x' = \gamma_u (v - u) t = \gamma_u (x - ut) \tag{4}$$

$$t' = \gamma_u \left[1 - \frac{v}{c}\cdot\frac{u}{c}\right] t = \gamma_u \left[t - \frac{xu}{c^2}\right] \tag{5}$$

where
$$\gamma_u = \frac{1}{\sqrt{1 - u^2/c^2}} \tag{6}$$

The ratio x'/t' of Eq. (4) to (5) gives (3). The corresponding formula for the hyperbolic tangent of a difference is given below.

$$\tanh(A - B) = \frac{\tanh A - \tanh B}{1 - \tanh A \cdot \tanh B} \tag{7}$$

Comparison of Eqs. (3) and (7) tells that:

$$\frac{v'}{c} = \tanh(A - B) \tag{8}$$

$$\frac{u}{c} = \tanh B \tag{9}$$

$$\frac{v}{c} = \tanh A = \frac{\sinh A}{\cosh A} = \frac{s\cdot\sinh A}{s\cdot\cosh A} = \frac{x}{ct} \tag{10}$$

where s is proper time.

For a stationary observer this gives

$$\frac{x}{c} = s\cdot\sinh A \tag{11}$$

$$t = s\cdot\cosh A \tag{12}$$

For a non stationary one there is

$$\frac{v'}{c} = \tanh(A - B) = \frac{\sinh(A - B)}{\cosh(A - B)} = \frac{s\cdot\sinh(A - B)}{s\cdot\cosh(A - B)} = \frac{x'}{ct'} \tag{13}$$

which gives

$$\frac{x'}{c} = s\cdot\sinh(A - B) \tag{14}$$

$$t' = s\cdot\cosh(A - B) \tag{15}$$

and finally the Minkowski formula [2] in the case of 1D space:

$$t^2 - \frac{x^2}{c^2} = s^2 = t'^2 - \frac{x'^2}{c^2} \tag{16}$$

In 3D space the Minkowski formula looks like this:

$$t^2 - \frac{x^2}{c^2} - \frac{y^2}{c^2} - \frac{z^2}{c^2} = s^2 = t'^2 - \frac{x'^2}{c^2} - \frac{y'^2}{c^2} - \frac{z'^2}{c^2} \tag{17}$$

which results in the formulae for velocity, time and distance shown in the next section.

3. Velocity Addition and Lorentz Transform Formulae in 3D Space

In such a case we have three different velocity components:

$$v'_x = \frac{v_x - u_x}{1 - v_x u_x / c^2}, \quad v'_y = \frac{v_y - u_y}{1 - v_y u_y / c^2}, \quad v'_z = \frac{v_z - u_z}{1 - v_z u_z / c^2}, \tag{18}$$

three different formulae for time:

$$t' = t\frac{1 - v_x u_x / c^2}{\sqrt{1 - u_x^2 / c^2}}, \quad t' = t\frac{1 - v_y u_y / c^2}{\sqrt{1 - u_y^2 / c^2}}, \quad t' = t\frac{1 - v_z u_z / c^2}{\sqrt{1 - u_z^2 / c^2}}, \tag{19}$$

and three different formulae for distances:

$$x' = \frac{x - u_x t}{\sqrt{1 - u_x^2 / c^2}}, \quad y' = \frac{y - u_y t}{\sqrt{1 - u_y^2 / c^2}}, \quad z' = \frac{z - u_z t}{\sqrt{1 - u_z^2 / c^2}}. \tag{20}$$

Following rules of vector calculus, the denominators in Eqs. (18) should equal one. In such a case, in order to obtain Galilean transform formulae from the Lorentz formulae, it is enough to set time t equal to time t'. As a result, the denominators in Eqs. (19) and (20) should also equal one.

4. Conclusion

Ratio of Lorentz distance to Lorentz time gives formulae for velocity addition, which do not obey the rules of vector calculus. According to Saa [3] the rules of four-vectors addition are the same as that for vectors in 3D space. Hence the introduction of the four-vectors does not solve the problem.

The conclusions are as follows

1. The silent assumption that the ratio of any velocity to velocity of light equals the hyperbolic tangent of an argument (rapidity) has been made against the rules used in science.
2. Consequently Einstein's formula for velocity addition does not agree with the correct formula for the addition of vectors
3. Minkowski formula and the Lorentz transform formulae are fully consistent with that erroneous Einstein formula hence all of them should be considered equally wrong.
4. Finally we propose to return to the Galilean transform formulae, which do agree with the rules of vector calculus.

5. References

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[3] Diego Saa, "Four-vectors in Electromagnetism", *Proceedings of the NPA* **5** (2): 218-233 (2008).

[4] W. A. Ugarow, **Specialnaja Teorija Otnositielnosti** (Nauka, Moskwa, Polish edition, PWN Warsaw, 1985)