Gravitational Light Bending History is Severely Impact-Parameter Dependent

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Recorded history of astronomical observations clearly shows that the rays of star light are gravitationally lensed solely the plasma rim of the sun, not in the plasma-free space for *impact parameters* of light rays above the plasma rim. Although the plasma atmosphere of the sun has a thickness of only a fraction of a solar radius, the gravitational light bending effect as predicted by the light bending rule of General Relativity should be easily detectable above plasma rim in the plasma-free space for several solar radii with current technical means. Findings show the solar plasma atmosphere represents an *indirect interaction* involving an interfering plasma medium between the gravitational field of the sun and the rays of the stars light. Calculations supporting this argument lead to the very same light bending equation obtained by General Relativity, derived from classical assumptions of a minimum energy path of a light ray in a plasma atmosphere exposed to the gravitational gradient field of the sun. This result is confirmed by some well cited researchers who used a very-long-baseline-interferometer (VLBI) measurement on extra galactic radio sources to determine the gravitational deflection of microwaves at the solar plasma rim, obtaining a result to within 0.9998 ± 0.0008 times that of General Relativity. Moreover, there is a clear lack of lensing among the countless numbers of stars, where likely candidates for gravitational lenses and point-like light sources are by good chance co-linearly aligned with the earth based observer. With this condition well at hand and by assuming the validity of the light bending rule of General Relativity, the sky should be filled with images of Einstein rings. Findings convincingly show that the failed observation of Einstein rings in the star-filled skies are fundamentally due to the **larger** *impact parameters* as can be understood from the geometry of the problem of the astronomical distances. The path of the observed light rays or microwaves will have to propagate in the plasma-free vacuum space significantly above the stellar atmosphere of the lensing stars due to the vast astronomical distances. The light bending rule of General Relativity suggests that gravitational light bending should occur in a plasma-free space as well as in the empty plasma-free vacuum space. The failed observation of the Einstein rings in the star-filled skies may be well explained by the absence of the *indirectly interacting* plasma media at high impact parameters above the plasma limbs of the sun and the stars.

Keywords: black hole, gravitational lensing, plasma atmosphere, galactic core, optical reciprocity.

1. Introduction

Close examination of the stars in our region of space and our sun suggest that the observed gravitational lensing is due to an *indirect interaction* .between gravitation and electromagnetism. The light bending rule of General Relativity assumes a *direct interaction* takes place between the gravitational field of the lensing mass, namely, the mass of the sun and the rays of starlight observed during events of the solar eclipses [1]. This case assumes the interaction takes place in a vacuum space free of plasma as well as in a plasma atmosphere. Historically, the gravitational lensing is observed primarily at the plasma rim of the sun. It is evident that the past century of the observed solar light bending was due to an *indirect interaction* between the gravitational field of the sun and the rays of star light. This argument is strongly supported by a calculation which derives the very same light bending equation obtained by General Relativity [1, 13]. The equation was derived from the assumptions of a minimum-energy path or a least-time path of light propagating in a plasma atmosphere exposed to the gravitational gradient field of the sun. **Appendix A** gives a detail calculation and a derivation of this famous light bending equation of General Relativity without using the assumptions of General Relativity [1, 13]. Lebach, et al [2] made VLBI observations on extra galactic radio sources to determine a gravitational deflection for microwaves at the plasma rim of the sun arriving at precisely an angular deflection of 1.75 arcsec at the solar plasma limb. The researchers reported a gravitational reflection of 0.9998 ± 0.0008 times that of General Relativity which confirms the calculation for the gravitational deflection of light in the solar plasma reported in this work and in References [1, 13]. Appendix A gives a detail calculation for the gravitational deflection of light. Examining the lower boundary of the solar atmosphere and the plasma-free vacuum space several solar radii above the rim of the sun, the solar light bending effect acting on the rays of starlight appear to deviate from the predicted $1/R$ effect of the light bending rule of General Relativity. A close examination of the stars in our own region of space, less than hundreds of light-years away, appear to exhibit the very same gravitational light bending effects as that of our nearest star, the sun. It is important to note that electromagnetic waves at microwave frequencies, on the order of cm wavelengths as opposed to the nm wavelengths of optical, infrared and ultraviolent waves, are at resonance with the solar plasma and the charge solar winds in the solar system. Microwaves are subject to a frequency dependent and impact parameter dependent time delay effect, commonly known as the *Shapiro Delay*; an effect which has absolutely nothing at all to do with the theoretical *space-time* effect of General Relativity. The *space-time* effect is independent of the frequency of the observed electromagnetic waves. Details are given in References [17, 18].

The observational evidence convincingly shows that *indirectly interacting* media containing gravitating matter cannot exist in the vacuum space at the site of Sagittarius A*. This is confirmed by the history of intense observations of the time resolved images collected from Sagittarius A* since 1992 [3-6]. Intense observations of collected images reveals to this date a clear lack of evidence for gravitational lensing or distortions as can be seen in the recorded images of the stars orbiting about the super massive object at the site of Sagittarius A* [3-7]. There are many cases in the star filled skies where the lenses and the light sources are by chance co-linearly aligned with the earth based observer, presenting vast opportunities for the observation of Einstein rings. For tutorial see **Appendix B**. We shall examine the collected images and the astrophysical data of the stars orbiting about Sagittarius A*, a region thought to contain a super massive black hole located at the center of our galaxy, the Milky Way, right in our own back yard, just 26,000 light-years away.

2. The Important Fundamentals

Application of Gauss's law of gravitation and the principle of optical reciprocity clearly show that a co-linear alignment of the observer, the lens and the source is unnecessary for an observation of a gravitational light bending effect, as predicted by the light bending rule of General Relativity. The gravitational effect at the surface of an analytical Gaussian sphere due to the presence of a point-like gravitating mass that is enclosed inside of the sphere depends only on the quantity of mass enclosed. The size or density of the enclosed mass particle is not important [9, 10]. The Gauss' law of gravity (see, e.g., [9, 12]) is a Mathematical Physics tool that encloses a gravitating mass particle inside of an analytical Gaussian surface which applies directly to the gravitational field of the enclosed mass. An analogy to this principle encloses an electrically charged particle inside of a Gaussian surface in application to the electric field of the charged particle in the discipline of Electromagnetism [12]. The principle of optical reciprocity [9, 10], simply states that the light must take the very same minimum energy path or least time path, in either direction between the source and the observer. This fundamental principle is an essential tool for the understanding of complex lensing systems in Astronomy and Astrophysics [12]. We shall now correctly apply all of these well founded and proven fundamentals to these gravitational lensing problems.

2.1. Gauss Law Applied to a Point-Like Gravitating Mass

Any gravitational effect on a light ray due to the presence of a gravitating mass at the impact parameter R would theoretically depend on the amount of Mass M that is enclosed within the analytical Gaussian sphere of radius R as illustrated in Fig. 1. Any gravitational effect that would be noted at the surface of the analytical Gaussian sphere should in principle be totally independent of the radius of the mass particle or the density of the mass that is enclosed within the Gaussian sphere. From Gauss's Law, Eq. (2), equal masses of different radii will theoretically have equal gravitational effects at the surface of the Gaussian sphere. The light bending rule

$$\delta\theta = \frac{4GM}{Rc^2} \tag{1}$$

of General Relativity is essentially a localized $1/R$ effect. Since we are dealing with astronomical distances, the impact parameter R is practically a localized effect only. The predominant gravitational effect would take place in the vicinity of the gravitating mass, namely the lens, maximizing at the point where the light ray is tangent to the Gaussian sphere of radius R.

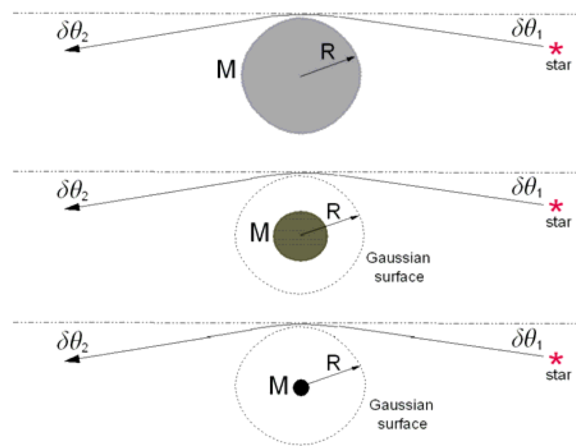

Fig. 1. Gauss' Law applied to Equal Gravitating Masses of Different Radii Enclosed

A very essential tool of Mathematical Physics known as Gauss' law [10, 12],

$$\int_S \vec{\mathbf{g}} \bullet d\vec{\mathbf{A}} = 4\pi GM \tag{2}$$

is applied directly to the gravitating masses where the gravitational field $\vec{\mathbf{g}}$ is a function only of the mass M enclosed by the spherical Gaussian surface S [9, 10, 12]. The gravitational flux at the surface of the analytical Gaussian sphere is totally independent of the radius R of the sphere. The idea here is that the gravitational field at this analytical Gaussian surface is only a function of the mass that it encloses [10,12]. Any mass M, regardless of the radius of the mass particle that is enclosed inside of the Gaussian spherical surface of radius R will contribute exactly the same gravitational potential at the Gaussian surface. In Fig. 1, the gravitational field points inward towards the center of the mass. Its magnitude is $g = \frac{GM}{R^2}$. In order to calculate the flux of the gravitational field out of the sphere of area $A = 4\pi R^2$, a minus sign is introduced. We then have the flux

$$\Phi_g = -gA = -\left(\frac{GM}{R^2}\right)\left(4\pi R^2\right) = -4\pi GM .$$

Again, we note that the flux does not depend on the size of the sphere. It is straightforwardly seen that a direct application of Gauss's law to the light bending rule, Eq. (1), coupled with the essential principle of *optical reciprocity* [11], removes any requirement for a co-linear alignment of the light source, the point-like gravitating mass particle (the lens) and the observer for observation of a gravitational lensing effect as suggested by General Relativity [12, 13].

From Eq. (2), the flux of the gravitational potential through the surface would be the same for all enclosed mass particles of the same mass M, regardless of the size of the mass particle. As a result, as illustrated in Fig. 1, each mass particle will produce the very same gravitational light bending effect $\delta\theta = \delta\theta_1 + \delta\theta_2$, where $\delta\theta_1$ and $\delta\theta_2$ are the bending effects on the ray of light on approach and on receding the lens, respectively. This of course assumes the validity of Eq. (1). This symmetry requirement suggests that $\delta\theta_1 = \delta\theta_2$, and from Eq. (1) that $\delta\theta = 2\delta\theta_1 = \dfrac{4GM}{Rc^2}$. It follows that $\delta\theta_1 = \delta\theta_2 = \dfrac{2GM}{Rc^2}$. This says that the total contribution of the light bending effect due to the gravitating point-like mass particle on any given infinitely long light ray is theoretically divided equally at the impact parameter R, separating the approaching segment and receding segment of the optical path.

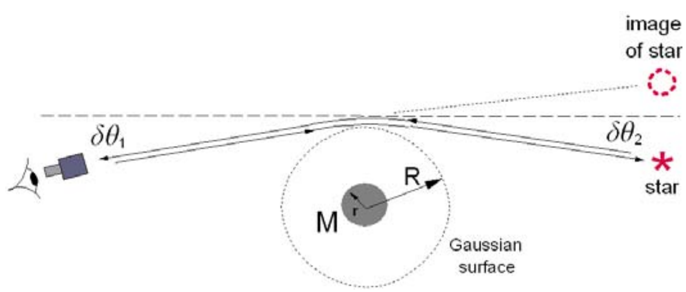

Fig. 2. Fundamental principle of optical reciprocity illustrated on a lensed light ray.

In any space, the principle of reciprocity [9, 11], a very fundamental principle of optics, must holds as illustrated in Fig. 2. The principle simply states that any photon or wave of light moving on a preferred optical path, from the source to the observer, must take the very same optical path from a hypothetical laser gun of the observer back to the source. As a consequence of this fundamental principle, any additional sources placed along the same preferred optical path will all appear to the observer to be located at the very same image position of the original source. As a consequence of this principle, all light emitting sources on a single preferred optical path will appear to the observer to be co-located at the very same point, appearing as a single light emitting source. This scarcely mentioned fundamental principle of optics is directly applicable to the Astrophysics at the galactic center. The total gravitational light bending effect acting on the light ray upon approach and upon receding a point-like gravitating mass is given by

$$\delta\theta = \left|\delta\theta_1\right|_{\substack{\text{approaching}\\ \text{the lens}}} + \left|\delta\theta_2\right|_{\substack{\text{receding}\\ \text{the lens}}} = \frac{4GM}{Rc^2} \qquad (3)$$

2.2. Principles of Gauss and Reciprocity applied to the Gravitational Deflection of Light and Microwaves

Assuming the validity of the light bending rule of General Relative, modern technical means of the astronomical techniques should have easily allowed observations of solar light bending of stellar light rays at different solar radii of analytical Gaussian surfaces, namely at the radius of $2R$, $3R$ and even at $4R$, where R is one solar radius, as illustrated in Fig. 3. At the analytical Gaussian surface of radius $2R$, General Relativity predicts an effect of one half the effect of 1.75 arcsec noted at the solar rim; at the impact parameter of $3R$, an effect of one third the effect at the solar rim, etc., etc. The equatorial radius R of the sun is approximately 695,000 km. The thickness of the solar rim is recorded to be less than 20,000 km; less than 3% of the solar radius R. Historically, the observed impact parameter for microwaves deflected at the solar limb was constrained to be exactly the solar radius R. However, at higher impact parameters greater than R, the deflections of microwaves and light are not seen above the plasma limb [2, 16, 17, 18]. Here, the light bending rule of General Relativity is clearly violated.

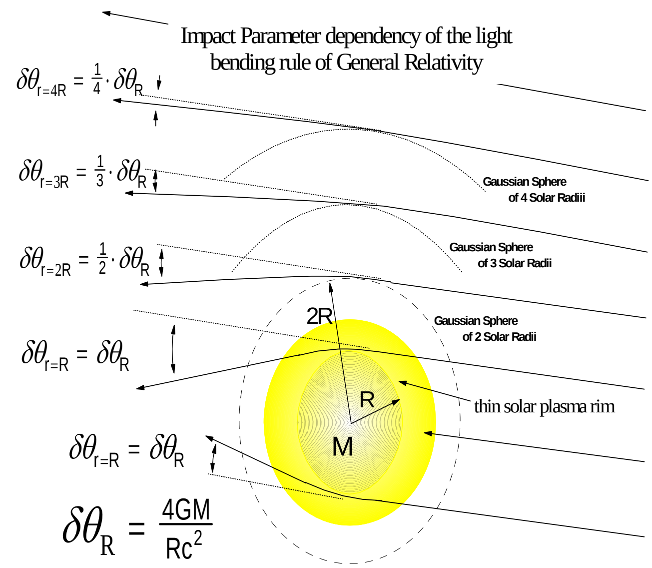

Fig. 3. Gauss' law applied to the gravitational light bending rule of General Relativity showing its theoretical impact parameter dependency.

2.3. The Fundamentals applied to the Orbit of S2 about Sagittarius A*

The past decades of intense observations using modern astronomical techniques in Astrophysics alone reveal an obvious lack of evidence for lensing effects on collected emissions from stellar sources orbiting about Sagittarius A*, believed to be a super massive black hole located at the galactic center of our Milky Way. This is most obviously revealed in the time resolved images collected since 1992 on the rapidly moving stars orbiting about Sagittarius A* [3, 4, 5, 7, 14]. The space in the immediate vicinity of a black hole is by definition an extremely good vacuum. The evidence for this is clearly seen in the highly elliptical orbital paths of the stars orbiting about the galactic core mass. The presence of material media near the galactic core mass would conceivably perturb the motion of the stellar object s16 which has been observed to move with a good fraction of the velocity of light. The presence of any media other than a good vacuum would have caused the fast moving stellar object s16 to rapidly disintegrate. Astrophysical observations reveal that s16 has a velocity approaching 3% of the velocity of light when passing to within a periastron distance corresponding to 60 astronomical units from Sagittarius A*, perceived to be a massive black hole. This gives solid evidence that the space in this region has to be, without a doubt, an extremely good vacuum.

3. Conclusion

Historically, the light bending effect has been observed primarily at the thin plasma rim of the sun. A detail calculation obtains the very same light bending Eq. (1) as that obtained by the light bending rule of General Relativity is found in Appendix A and in References [1, 13], only this time Eq. (1) applies directly to the bending of light rays in a plasma atmosphere exposed to the gravitational gradient field of the sun. The calculated results of this research is confirmed by Lebach, et al [2], who used VLBI techniques on extra galactic radio sources to determine the gravitational deflection of microwaves at the solar plasma rim. An angular deflection of exactly 1.75 arcsec for microwaves was also observed by a number of other researchers who obtained impact parameters corresponding to the very thin plasma limb of the sun. Higher impact parameters for microwaves from quasar radio sources or artificial satellites are yet to be observed. The higher impact parameters would have resulted in reduced angular deflections less than 1.75 arcsec as is predicted by the light bending rule of General Relativity. The extended history of microwave deflections of 1.75 arcsec observed at the solar limb using VLBI techniques confirms that an indirect interaction between microwaves and the gravitational gradient field of the sun actually takes place. The thickness of the very thin solar plasma limb constrains the impact parameter for microwave deflections to be only slightly larger than the solar radius R. Findings show that a *direct interaction* between the sun's gravity and the rays of star light in the empty vacuum space is yet to be observed. The celestial skies present vast opportunities to modern Astronomy and Astrophysics to allow for easy detection of gravitational lensing effects, as predicted by General Relativity, due to the large numbers of stellar objects that happen to be co-linearly aligned with the earth based observers. This, course, assumes that the light bending rule of General Relativity applies to the plasma-free space as well as to the plasma atmosphere of the sun and stars. Because of the vast astronomical distances between the stars, gravitational lensing of light rays must have impact parameters such that they pass clearly above the plasma rim of the lensing star. With application of the important fundamentals, the observations clearly reveal that only *indirect interactions* actually take place between the gravitational field of the sun and the rays of microwaves and the rays of star light. Thus, it is of no surprise that the Einstein rings are not observed in the star-filled skies. For tutorial see **Appendix B**. The very same fundamentals apply directly to all celestial skies and to the events taking place at Sagittarius A* [1, 3, 4, 5, 6, 7, 15]. The evidence is clearly in the everyday cosmological appearance.

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A. Bending of Electromagnetic Waves in Solar Plasma Limb as function of Gravitational Potential; a Minimum Energy Path Calculation

A calculation for the bending of light rays in the thin plasma rim of the sun is carried out in detail here. The calculation is based entirely on a conservation of energy concept considering the gradient of the gravitational field of the sun acting directly on the rapidly moving ionized material particles of the thin plasma atmosphere of the sun. The calculation considers a minimum energy path for rays of light. The result is found to be totally independent of frequency. The rapidly moving ionized particles of the solar plasma is assumed to be bounded by the gravitational potential of the sun given by

$$\phi_{r=R}^{r=\infty} = \int_{r=R}^{r=\infty} \frac{GM}{r^2} dr = \frac{GM}{R} \tag{A1}$$

It may be assumed that the plasma particles of the ionized solar rim move with random velocities such that their kinetic energies are as dictated by $\frac{1}{2}mv^2 = \frac{3}{2}kT + \phi m$, where **m** is the mean mass of the plasma particles of temperature K°, v is the velocity of the plasma particle bounded by the gravitational potential ϕ. The velocity v of the moving ions may be assigned an upper bound of $v = \sqrt{\frac{2GM}{R}}$, the escape velocity of the solar gravity at the surface of the sun. The solar plasma particles bounded by gravity in the solar rim may be considered as a dynamic lens under the intense gravitational gradient field of the sun. It is theoretically shown here, and in detail in [13}, that a minimum energy path for light rays propagating in the solar plasma rim, subjected to the gradient of the gravitational field of the sun, yields the mathematical results of $\frac{4GM}{Rc^2}$.

It is shown that the moving ions acting as secondary sources within the plasma rim, moving with velocities not to accede the velocity $v = \sqrt{\frac{2GM}{R}}$, the frequency and wavelength of a light ray exposed to the plasma are:

$$\nu' = \nu_0\left(1 - \frac{v^2}{c^2}\right) = \nu_0\left(1 - \frac{2GM}{Rc^2}\right) \tag{A2}$$

$$\lambda' = \lambda_0(1 - \frac{v^2}{c^2})^{-1} = \lambda_0(1 - \frac{2GM}{Rc^2})^{-1} \tag{A3}$$

$$\lambda' \approx \lambda_0(1 + \frac{2GM}{Rc^2}) \tag{A4}$$

From this a reddening of the frequency and a slight lengthening of the wavelength occurs. From this, the number of wavelengths per unit length along a minimum energy path for the light ray propagating within the plasma rim may be given as

$$n = \frac{1}{\lambda'} = \frac{1}{\lambda_0(1-\frac{v^2}{c^2})^{-1}} = \frac{1}{\lambda_0}(1 - \frac{2GM}{Rc^2}) \tag{A5}$$

Thus, the energy ε per unit length of the light ray along the minimum energy path is $\varepsilon = \varepsilon_0(1 - \frac{2GM}{rc^2})$. Consequently, the number of re-emitted waves per unit length along the photon path and thus the energy per unit length increases as r increases. This translates to a downward, re-emitted path of the bent light ray, along a minimum energy path for the approaching segment of the light ray. If $\frac{d\varepsilon}{dr} = +\varepsilon_0 \frac{2GM}{rc^2}$ or $\delta\varepsilon = +\varepsilon_0 \frac{2GM}{rc^2}\delta R$, then the reemission of the light ray in the atmosphere of ions will occur such that the total energy along the minimum energy (conservation of energy) path for a given light ray would not change. If ε is the energy per unit length along the light ray and $\delta\varepsilon$ is the change in energy in the direction of the gradient potential $\phi(r)$, then the angle of change during the approach segment of the light ray is

$$\delta\theta_{app} = \frac{\delta\varepsilon_{app}}{\varepsilon} = +\int_{r=\infty}^{r=R} \frac{2GM}{r^2c^2} dr = -\frac{2GM}{Rc^2} \tag{A6}$$

and the path change for the receding segment of the light ray is

$$\delta\theta_{rec} = \frac{\delta\varepsilon_{rec}}{\varepsilon} = +\int_{r=R}^{r=\infty} \frac{2GM}{r^2c^2} dr = +\frac{2GM}{Rc^2}. \tag{A7}$$

The net change in the path of the light ray is

$$\delta\theta = \delta\theta_{rec} - \delta\theta_{app} = \frac{4GM}{Rc^2}. \tag{A8}$$

B. The Einstein Ring Equation; the General Case ($D_L \neq D_{SL}$)

The general case for the Einstein ring equation involves all values for the distances, whereby D_L is the distance between the observer and the lens and D_{SL} is the distance between the lens and the source. These are cases where $D_L \neq D_{SL}$. The general case for the radius of the Einstein ring in units of radians is

$$\delta\theta(rad) = \sqrt{\frac{D_{SL}}{D_L + D_{SL}} \frac{4GM}{D_L c^2}} \tag{B1}$$

The radius of the Einstein ring at the image location the distance of $D_L + D_{SL}$ expressed in meters is

$$R(meters) = (D_L + D_{SL})\delta\theta(rad) \tag{B2}$$

where D_L and D_{SL} are also expressed in meters. The impact parameter (IP) corresponding to the Einstein ring is the nearest point of approach of the light rays to the point-like lensing mass, when observed at a distance of D_{SL} meters away from the observer, for the rays of light coming from the light source to the observer. Since this is a 3 dimensional problem, the impact parameter of the light rays that would produce an Einstein ring is also a ring itself. It is a virtual ring for purpose of the analysis of the problem. The impact parameter (IP) in meters is

$$R_{IP}(meters) = (D_L)\delta\theta(rad) \tag{B3}$$

where R_{IP} is the nearest point of approach of the gravitationally lensed light rays to the lensing star. It is that distance the lensed light rays will pass over the plasma rim of the lensing star, moving through the empty vacuum space well above the plasma rim of the lensing stars, moving along astronomical distances from the source to the observer. The radius of the predicted Einstein ring, according Eq. (B1) and the light bending rule of General Relativity, will be nearly 15 times the radius of a sun-like lensing star, the same mass and radius as the sun, when both are observed at the same distance $D_{SL} = 4$ light years away. Adjusting the parameter D_{SL} would cause the radius of the Einstein ring to change. Increasing D_{SL} would cause the image of the Einstein ring to increase in radius (an increase in magnification), assuming the validity of General Relativity. Setting $D_L = D_{SL}$, Eq. (B1) becomes $\delta\theta(rad) = \sqrt{\frac{2GM}{D_L c^2}}$, the special case.