Ending Quantum Physics' Dependence on Copenhagen Doctrine

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Bridging the gap between quanta and the rest of physics a thorough awareness of the following errors in judgment of contemporary interpretive schemes is a sine qua none. The first error was and is a totally unfounded assumption that Schrödinger's Ψ function might be describing a single quantum system; all other errors are contingent on this one. The second error became concocting a nonclassical statistics as a futile attempt at covering up Bohm's hidden variables as conceivably accounting for any changes in statistics. The third error is a related existence of an ever-present statistics of quantum uncertainty postulated by Heisenberg. It was derivable from Schrödinger's equation thus logically forcing: The fourth error is an ensuing endowment of Schrödinger's equation with an unwarranted first principle status. These four epistemic errors are avoided by replacing the single system by an ensemble of identical systems subject to a perfectly classical correlation statistics. Schrödinger's Ψ now deals with optimal ensemble randomness of minimal correlation reflecting the state of its spectral sources. Without the man-made, ever-present, non-classical statistics, a pre-statistical quantum order can now be resuscitated. It calls for tools capable of probing topological order. Avoiding injecting prejudicial information, its tools need to be topological invariant assessing global order. The Aharonov-Bohm integral and its two- and three-dimensional companions indeed meet those requirements. They are here discussed and tested by resolving a persistent thirty year old quantum Hall dichotomy.

1. The Early Days of Quantum Recipes

The beginning of the quantum realm in physics is invariably associated with Planck's act of removing infinities from the then existing radiation law of Raleigh-Jeans. He did so by making the stunning assumption that radiation energy existed in finite quanta which he took to be multiples of a unit hv in which h was taken to be a new constant of nature and ! a frequency somehow associated with that quantum unit. Unlike the Raleigh-Jeans law, an integration of this new law over the total frequency realm from 0!" gave finite results for the Stefan-Boltzmann law of total heat radiation while providing very relevant information about the structure and value of the Stefan-Boltzmann constant.

Five years later Einstein gave Planck's initial proposition a thumps-up with his explanation of the photoelectric effect. Then in 1911 Bohr introduced angular momentum quantization in terms of multiples of h as a counterpart of Planck's initial energy quantization. The latter proposition opened up the atomic realm to quantization with an avalanche of applications of historic proportions. They led to an understanding of the periodic system of chemical elements in terms of electron shells. Numerous spectral regularities could be understood and hydrogen-like structures responded to the developing theory.

Matching atomic mechanics with relativity, Sommerfeld revealed quantitative correct data of spectral fine structure. At about the same time Einstein began viewing existing radiation density as a balance between induced- and spontaneous emission and absorption, enabling him to come up with a transparently simple derivation of Planck's radiation law.

So by the beginning of the Twenties the quantum realm appeared well on its way of becoming an integral part of modern physics. Yet it seemed curious that this wealth of new information seemed to be emanating from two almost primitive looking propositions:

Planck's multiples of energy units hv and Bohr's angular momentum multiples h. In the here cited cases h appeared as a common key of what today is called quantization.

Yet, in time discomfort developed about this rapidly developing quantum formalism depending on an unpredictable display of discreteness for some entities and not others. In global mathematics a similar kind of distinction exists between closed an exact differential forms. Its relevance to quantization will be taken up here, yet prior to this, global concepts, as distinguished from local concepts, were beginning to make their presence known in a new branch of quantum mechanics of the mid Twenties.

2. The Heisenberg-Schrödinger Eigenvalues

Heisenberg initiated a fuzzied orbital picture of the quantum process that led to quantum states as algebraic eigenvalues, soon followed by Schrödinger with an analytic eigenvalue process. The two procedures were proven to be equivalent by Schrödinger himself and by Pauli. The Schrödinger process prevailed, because the algebraic process was unwieldy, whereas the analytic process could take advantage of a very penetrating and elegant 1924 Courant-Hilbert treatise [1] dealing with its ramifications.

The eigenvalue process is global in nature, because its eigenvalues are constants associated with a given ground domain. In the Courant-Hilbert case that is normally determined by conditions cited for the boundary of that domain. Boundary conditions of Schrödinger's eigenvalue require its Ψ function to be single valued and square integrable over the ground domain, which is all of space! Copenhagen Doctrine holds Ψ to be a probability density, somehow pertaining to a nonclassical statistic behavior of the presumed single system. In 1934 Popper [2] assigned an ensemble basis to Ψ, yet leaving the meanwhile accepted Copenhagen-basis as conveying a nonclassical probability density.

3. Classic Ψ Statistics Casts Light on Popper Ensembles

For Copenhagen's single system interpretation, the integration domain covers all of physical space. For the ensemble alternative it only needs to cover the finite domain of the ensemble source, which gives us an opportunity to become more specific about the ensemble statistics. Explicit ensemble averaging procedures by Planck and others had yielded Schrödinger eigenvalues $(n+1/2)h\nu$ for energy and $h\left[n(n+1)\right]/2$ for angular momentum.[1]

For many practical purposes this option is the same as Copenhagen's proposition to consider all of physical space, because it covers that part of space that physically counts i.e., the experimental ensemble domain of the randomized spectral source. The Ψ function declines very steeply compared to the size of most spectral ensemble sources. It secures optimal randomness of phase and orientation in finite ensemble sources, beyond that source there are no ensemble elements, i.e., nothing to correlate in the ensemble.

Another basic distinction between the Schrödinger- and the Courant-Hilbert type eigenvalue processes is that the Courant-Hilbert process deals mostly with resonance systems of waves ordered in mutual phase and orientation. In fact, phase and orientation provide the distinctive acoustic coloring of the great variety of musical instruments. Composers develop unusual expertise in such matters for purposes of orchestration and musical audiences have become sensitized to respond to the near-endless varieties of acoustic mixing presented to them in time sequences of changing pitch.

While the Courant-Hilbert process covers a finely tuned musical ordering of mutual phase and orientation, Schrödinger's process deals with (fairly large) ensembles of identical (atomic or molecular) sources as optimally randomized resonance entities. Their state of complete phase- and orientation disorder is their state of normalcy. This is totally unlike the Courant-Hilbert process manifesting itself as those subtle optimal states of phase orientation covering an infinite gamut of musical coloring.

Briefly stated, Courant-Hilbert approach refers to an order optimum, Schrödinger refers to a disorder optimum. The ensemble nature of its spectral source indeed adds a hint of disorder for Schrödinger's assessment. It is not quite clear how the same process tool might be capable of changing itself to deal with a continuous transition to ensemble order. That transition calls on drastic changes in the tools of description.

Copenhagen's nonclassical statistics, implying the existence of an always-present zeropoint energy, and the uncertainty thereof, had already preempted the above conclusion by somewhat rudely excluding the very existence of order alternatives. Copenhagen sadly failed accounting for making nature all statistic! The next section covers a chapter of a near-forgotten period of breakthrough evidence of a truly existing quantum order in theory and experiment, thus ruling out this nonclassical as just far too nonclassical.

4. The Ordered Quantum Alternatives of the Sixties

Pre-statistic quantum phenomena were around in the early days of quantum receptology, ironically they were the ones that led to Schrödinger's equation when the latter became an epitome of statistic quantum aspects. Monochromatic (speed-ordered) electron rays exhibit selective reflection from lattice surfaces in crystals. Davisson-Germer [3] and father and son Bragg [3] pioneered that area. Yet the fact that these developments played a distant role in the birth of Schrödinger's equation as a statistic tool may have obscured these effects as prototypes of ordered pre-statistical phenomena.

When Born and Heisenberg announced the Copenhagen Doctrine in1927 at the Solvay meeting in Bruxelles, it initiated a trend of denying its pre-statistic origin by placing Schrödinger's equation on an independent statistic level. They even designed a priori non-classical statistics for that purpose, a statistics made by man not by nature.

Later in the century people considered interference between electron beams and self interference of beams. This led Aharonov-Bohm [4] in 1959 to a space-time line-integral of the four-vector potential being equal to multiples of the flux quantum h/e accounting for an interference experiment by the same two authors. Two years later Doll et al [5] and Fairbank et al [6] independently showed that little superconducting rings had fluxes emanating from them equaling multiples of half the Aharonov-Bohm flux, that is $h/2e$. In 1962 the Josephson [7] junction experiments exhibited $h/2e$ quantization also for the electric potential companion of the Doll-Fairbank magnetic potential flux. The dual manifestation of electro- and magneto fluxes matches such expectations as covered by the Aharonov-Bohm integral. Even so, these closely related quantum phenomena are still today oddities.

A step at revealing a wider context of these findings was made by Kiehn's [8] focus on Gauss' integral of electrostatics and the space-time generalization thereof. While Aharonov-Bohm was identifiable as a counter of flux units $h/2e$, the space-time Gauss integral can be taken as a counter of a net number of charge units; that evolution was realized by Faraday's electrolytic experiments of 1835 identifying the existence of a smallest charge unit e. Later in the 19th century, it became clear that this result was universal. Summarizing these results, the 1-dimensional Aharonov-Bohm integral is a counter of flux units $h/2e$ and this 2-dimensonal Gauss-Faraday integral is a counter of net charges e.

[1] These classic supports of the ensemble alternative have been reproduced in ref.9 see Planck, Kompaneyets

These statements have a topological connotation in that both integrals have cyclic integration domains. The one-dimensional AB integration is seen as linking closed flux loops and the two-dimensional GF integration can be viewed if you will as a ring surface enclosing a super-current circulating in a ring. The GF integral equals the number of charges participating in the super-current inside the integration ring as a multiple of e. With the AB and GF integrals accounting for quanta $h/2e$ and net quanta e a next question is whether Planck's original action quantum has an integral of his own.

Kiehn [8] has suggested a 3-dimensional product formation of the AB and GF integrals as action integral; calling for closed cyclic integration domains of 3-dimensions. The latter by necessity call for a space-time imbedding, giving general quantization a space- and time-like connotation. The three integrals in differential form notation are now:

Aharonov-Bohm $$\oint_{c_1} A = n\frac{\tilde{h}}{2\tilde{e}}; \qquad n = 1,2,3... \qquad (7)$$

Gauss-Ampère $$\oiint_{c_2} \tilde{G} = s\tilde{e}; \qquad s = 1,2,3... \qquad (8)$$

Kiehn product $$\oint\!\!\oint\!\!\oint_{c_3} A \wedge \tilde{G} = ns\tilde{h}; \qquad n = 1,2,3... \qquad (9)$$

The differential one-form A is defined by relativity's 4-potential, the Maxwell fields **D** and **H** define the 2-form !G , tildes specify impair behavior (i.e., sign change under spacetime orientation change of reference).

Two of the integrals have had at least a somewhat unofficial role in physics over the past half century for the simple reason that Schrödinger's process cannot handle well the ordered situations that do well by integral application. Except the just cited explicit experimental confirmations of Eq. (1) provided by [5, 6], Eq. (2) has been a trusted part of Maxwell theory. The implication is any other successful applications of these integrals puts the traditional Copenhagen view more on notice that something is amiss in Copenhagen's original claims. Presently the most notable applications of these integrals pertain to the Josephson- and quantum Hall effects. While these applications have been published in the open literature [9] there have been no responses to speak of to these realistic and simple alternatives. The deeper hurdles still standing in the way of admitting integrals (1,2,3) as honorable members of the physics family have their origin in the Copenhagen Doctrine.

5. Quantization as a Pre-Metric Experience

During the first half of the 20th century, physics had gone through a traumatic experience in their efforts of rewriting the Dirac equations into a generally invariant form. It was an idea of creating a bridge between the special-relativistic Dirac equations and the general theory of relativity. The best minds in physics and mathematics gave it a very serious try, yet the end result became a set of horrifying equations without a hint whatsoever of solutions offering a hopeful perspective of a substantive new physical insight. One of the critical new insights provided by general theory of relativity was how gravitational and kinematic accelerations both came out of the metric structure of the Christoffel symbols. Dirac spinor insights into space-time orientation helped the special relativistic realm, adding space-time curvature though then began clouding these extended efforts at quantization.

This disastrous development created an atmosphere of distrust of the quantum folks against those over-preoccupied with relativity General covariance became suspect in quantum circles, even so as we shall see soon, only a more discriminating view of covariance could improve matters. Fear of relativity's superhuman complexity sustained distrust. It is against this backdrop of earlier experiences that a vast majority of physics responded with a semi-conscious distrust against the integrals (1,2,3). It should be mentioned these integrals are indeed general invariants, however, it is easily proven no Christoffel symbols are needed to establish this honest to goodness general invariance. This qualifies its invariance as belonging in a more special higher category of topological invariance, unlike the basics of the general theory of relativity because it exists in a metric-based world.

Whereas SR(3) invariance is good enough for Schrödinger-type quantum mechanics and L(4) for covering Dirac theory, the pre-metric integrals [1,2,3] are covered by general transformation G(4) including space-time orientation changes. Here is a quantum realm in need of an upgrading from SR(3) to G(4) in a pre-metric realm. This change may help in swallowing unfriendly words against relativity mathematics after the disheartening experiences with transcriptions of Dirac's equations in the metric domain.

It should now be clear that the metric-free invariance of the integrals is global in nature, they convey a domain validity characteristic of topological situations. In fact the three integrals [1, 2, 3] have now been proven to fit requirements of a mathematical discipline that has been in existence since the Thirties and is now known as de Rham's Cohomology [11]; it probes topology using cyclic (period) integrals. In the current physical context, integrals {1,2,3} become tools probing pre-metric structure of the Maxwell fields.

In 1924 Cartan [10] identified the integral form of these pre-metric structures. De Rham, another member of the French school, then elaborated on using such non-vanishing cyclic integrals for topological exploration in 1931.[2] It means the closed 1-form A and closed 2-form !G reveal presence of flux and charge and the discreteness thereof. The integrals (1,2,3) are special cases of de Rham's theorem pointing at the existence of finite residues, also called periods. Experiment reveals their sizes solely as functions of e and h.

A quantum superstructure of Maxwell theory interacts here on an indisputably fundamental level with discrete principles of topology. From Bohr's recipe of angular momentum quantization to Sommerfeld's energy momentum integral, the Aharonov-Bohm- and Gauss-Faraday field integrals emerge as a grown up versions of the original Bohr recipe.

Consider that an electron circulating in a constant magnetic field B in its n^{th} quantum state intercepts a quantum flux $nh/2e$.

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[2] A circle of involved algebraic topologists at LMU is hereby gratefully acknowledged for having strengthened confidence in extrapolating application of de Rham's famous theorem to the cited physical cases.

Just substitute the cyclotron frequency $\omega = (e/m)B$ in the Bohr condition, $mr^2\omega = nh/2\pi$, the flux intercepted by the orbit $\pi r^2 B = nh/2e$ as indicated by the AB integral. This unexpected alignment between global flux quantization, Gauss' law and the Bohr condition indicates for the latter a measure of pre-statistic exactness.

6. The Pre-Statistic Pre-Metric Integrals

The three odd integrals collected by Kiehn are now part of a family that meet de Rham's condition of a complete set for assessing topological situations. One integral dates back to 1835 after Faraday identified the existence of a smallest electric charge e, permitting the conclusion that the Gauss integral of electrostatics should be regarded as a counter of net charge. The existence of a flux counting integral became a solid fact of life more than a century later in the Sixties of the last century through the collective efforts of Doll [5], Fairbank [6], Aharonov-Bohm [4] and Josephson [7].

Sommerfeld and others had earlier used a 1-dimensional action integral of mass-point momentum as a counter of multiples of Planck's quantum of 1900. However, the masspoint abstraction used in the Sommerfeld quantum counter prevents it from being a field integral as required by de Rham [11] theory. Kiehn [8] has considered the option of taking the 3-dimensional integral Eq. (3) as a product version of Eqs. (1) and (2), equaling action and angular momentum as the product of two quantum numbers s and n times its smallest elementary unit $h/2$. Except perhaps for undecided factors of 2, the three integrals Eqs. (1,2,3) not only exhibit a striking resemblance to a complete de Rham set of period integrals, the resemblance becomes an identification by the accurate measurements of e and h sizes. The quantum Hall effect in conjunction with Josephson's effect, have yielded the most consistent high precision values to date of the quanta e and h.

The quantum Hall resistance (impedance) ZH is defined as the ratio of the Hall potential V and the longitudinal Hall sample current I, yielding the following options: for n counting flux units $h/2e$ and s counting Cooper pairs of charge $2e$ per cyclotron.

$$Z_H = \frac{\text{integral }(1)}{\text{integral }(2)} = \frac{n}{s}\frac{h}{4e^2}, \quad n = 1,2,3\ldots \quad s = 1,2,3\ldots \qquad ()$$

The standard empirical formula has instead n flux units h/e and s charge units e.

$$Z_H = \frac{\text{integral }(1)}{\text{integral }(2)} = \frac{n}{s}\frac{h}{e^2}, \quad n = 1,2,3\ldots \quad s = 1,2,3\ldots \qquad ()$$

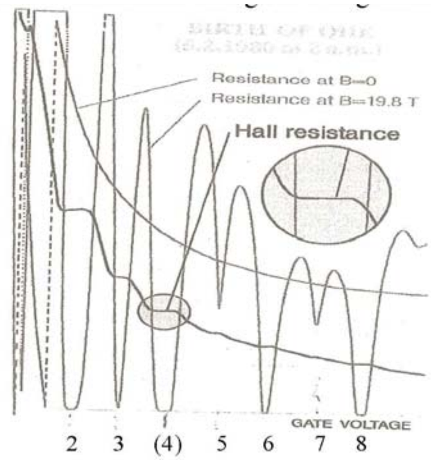

Fig. 1. An annotated version of the Klitzing data reproduced from Fig. 2 in [12], with (4) as identified filling factor, with added data {2,3,(4),6,7,8}. Resistance dips at even data reveal a conceivable Cooper pairing.

The discrete n and s values in I an II mark the same plateau states shifted by a factor 4. The n/s ratios taken from the literature are all reduced fractions assumed as corresponding to II as displayed above. The metrologist's use of formula II is de facto proof of their confidence in ZH as discrete flux over discrete charge. Hence, while not quite covered by prevailing theory, option II adds evidence of flux and charge as being individually quantized.

The cited reduced simple filling factors correspond to a charge carrier structure, taken to be a cyclotron formation, say with the real s charge units orbiting the real n flux units.

The original observations of 1980 are displayed in Fig. 1 of ref. [12]. Von Klitzing uses the gate voltage of his transistor sample for injecting charge in its 2-dimensional interaction space. His observations indicate how the sample current alternatively goes through a near superconducting state followed by a state of not so perfect conductivity. This could be indicative of a Cooper pairing role for the more perfect plateaus. A change of emphasis on getting real n/s ratios versus the cited reduced values would call for an independent n or s evaluations for given plateau ratios n/s.

7. General Covariance Separates Apples and Oranges

Einstein initiated his principle of general covariance to launch his general theory. It was in essence a logical continuation of the grammar school warning of not adding apples and oranges. If you don't separate apples and oranges you do get bigger numbers but you know less as to what these numbers mean. For more meaningful numbers we learn how to distinguish between apples and oranges.

Translated into the language of general covariance, it means the language of physics in a Riemannian space-time has a potential of being more perceptive of distinctive features that don't show up in a Cartesian 3-space description against absolute time. The in-between station of Lorentzian space with an indeterminate metric + - - - opens up new kinematics. The general theory though claims experimental evidence of gravity as having a metric connotation for a Riemannian space-time with an indeterminate metric. It means learning to differentiate between wider arrays of physical options by subtly loosening longstanding ties to Euclidean-Galilean prejudice. Einstein's realm of nonlinear reversible Diffeo (4) space-time changes greatly enhanced our perspicuity of viewing relevant physical distinctions. The same principle shall now be used by adding the claim that quantization calls in addition for space-time topological distinctions.

Since topological criteria are known by their invariance under deformation, it makes sense seeking metric-independent criteria revealing quantum-topology connections. Concentrating on metric-independent criteria separates the quantum realm from the realm of the general theory of relativity. Unbeknown to most, there is a realm of physics that indeed is independent of the Christoffel symbols of relativity carrying the acceleration plus gravity information. In the early 1930s Kottler, and Cartan established Maxwell's preconstitutive equations to be generally invariant, independent of the Christoffel symbols and that means independent of changes in the metric for that matter.

These metric-independent features have been reexamined in a Dover reprint [13] and the reason for bringing them up at this point is that the pre-statistic quantization integrals {1,2,3} constitute a set of domain-invariant statements. These integrals are topological domain invariants in the sense of de Rham theory. The proof is simple and not shown here. The metric has no role in these statements {1,2,3} they are 'size' independent, i.e., they have topological stature over and above the metric-based statements of relativity.

Those rubbed the wrong way by indices occurring in tensor expressions invoked by general covariance may now get guidance from some dimensional analysis. Replacing the mass dimension m in the standard reference {*m*, charge, length, time} by an invariant action unit say *h*, yields the new reference system becomes {*h*, *e*, *l*, *t*}. The core set {*h*,*e*} of this basis are invariant and the units {*l*, *t*} guide the transformation: *l* gives a contravariant superscript index, and $l-1$ gives a covariant subscript and similarly *t* and $t-1$ for time index. Just try it to like how this topology feature in dimensional analysis yields compelling notational recipes for physical tensors. Tensor densities of weight ±1 add a common dimensional factor{l±3t±1}; Schouten's alternating unit tensor [14] densities give dual transcriptions between co-and contravariant without affecting physical dimensions!

The here cited invariant delineation of dimension centers on physical application to tensors and differential forms, based on the recognized invariant status of the quantization integrals {1,2,3}. An incorporation of these matters in general physics can contribute much in clarifying the currently rather obscure perspectives between quanta and relativity.

Having referred to the work of de Rham a few words are in order how this major chapter in pre-metric manifold theory came about. Early methods of mathematically assessing topological structure go perhaps back to the Euler index of geometric configurations by covering them with a triangulation net. The alternating sum of points, line segments and enclosed surface elements add up to the Euler index as a structural invariant independent of the fineness of the triangular subdivision. This era of Betti number-characterization of geometric structures was followed by a more general procedure in which points, line-segments and surface patches were replaced by a dimension-independent type of distinction between boundaries and cycles. Point pairs bound line segments, paired circles bound a cylindrical surface and, if you will, r-cycles together constitute bound a totality of r-holes in a spherical surface. Hence boundaries need added specificity over cycles. The boundary versus cycle distinction became the basis of Poincaré's Homology discipline in n-space.

Mindful of the *p* versus *n-p* dimensional duality, Poincaré Homology gave rise to a Cohomology counterpart. In the beginning of the 20th century this duo together led to a discipline now called algebraic topology. While the homologies were in essence an outgrowth of algebraic geometry, which in turn had emerged from analytic geometry, at this point a contact with a parallel development in differential geometry had still been missing. In the early Thirties Swiss mathematician Gorge de Rham pioneered a joint homology-cohomology approach using general-invariant scalar- or pseudo scalar-valued forms.

De Rham made cyclic integrals a central theme in his topological assessment and in this light a contact with physics emerges in terms of the one-dimensional Aharonov-Bohm and the 2-dimensional Gauss-Faraday integrals. The A-B integral is an absolute scalar whereas the G-F integral is a pseudo scalar. The invariance realm encompasses general invertible space-time substitutions with Klein's vierer sub group for orientation change.

Differential p-forms integrated over cyclic p-domains are absolute- or pseudo scalars. They become period integrals if(f) the p-cycle or p-cycles link or enclose domains were the differential form differs from zero such that the p-cycle of integration resides everywhere in domains where the differential form vanishes. The value of the integral is called the period or residue. The differential form dictates how the p-cycles are bounding.

A differential form of all zero periods is said to be exact. Differential forms with nonzero periods are said to be as closed. De Rham theory defines the cohomological criteria exact versus closed of forms as dual counterpart of the homological assignments denoted as cycle versus boundary. De Rham theory is therefore a cohomology approach in the sense that the differential form, i.e., the anti-symmetric tensor fields defining the form, determines the homology of integration cycles. It is in this matching of homology features to the cohomology structure of the defining anti-symmetric covariant field where de Rham theory invites application to appropriately chosen pre-metric physical fields.

Since a closed p-form derives from an exact p+1 form, the pair makes the existence theorem of de Rham a matter of dimensional induction. The existence of all three integrals {1,2,3} in section 4 then follows by induction; from a proven existence of the Gauss' integral {2}, all are contingent on the viability of the p+1 conservation laws for p=1,2,3. De Rham existence theorem on mathematical discreteness closes in on its physics counterpart!

8. Conclusion and the QH Dichotomy

Ending the rule of Copenhagen Doctrine as guiding principle for the quantum realm opens up considerable possibilities of mathematical rationalization. It leads to an avalanche of new physical perspectives vying for attention. The emergence of pre-statistical global quantization, for one, calls for an introspective foundations reassessment of Schrödinger's process, while opening up a new wide area of topologically probing of all kind of ordered quantum structures, ranging from low temperature man-made cyclotron configurations all the way to what would have to be a new understanding of the order of atomic structures that seem to be truly surviving at considerably higher temperature. Here we have been paying the price of rudely bereaving Copenhagen from using its nonclassical escape!

A confession is necessary about Schrödinger's process. The transition from single system to an ensemble of identical single systems means the process reports about average 'identical' behavior in ensembles. The original Copenhagen Doctrine, by contrast, has been taken as reporting about how given single systems interact with a postulated non-classical background level. The Doctrine injected a false depiction of a mystical interaction with the single system mechanics, whereas it was the ensemble holding the center of attention.

The unfounded all-statistic quantum dogma in the wake of Copenhagen Doctrine has prevented solid-state specialists from using the global medicine of Eqs.1,2,3 on one of the most spectacular pre-statistical manifestations in the form of the quantum Hall effect. First attempts at understanding were made through Schrödinger's process. The ensuing artificial integer-fractional dichotomy so encountered was a man-made attempt at escape; what could be expected from using Schrödinger's statistic equation on one of the most well-ordered pre-statistic phenomena known to date: i.e. the Q.H. effect?

This perhaps robot-type decision-making by following dogma calls for radical interpretive change. Copenhagen's single system needs to be changed into an ensemble reality with natural classical statistical parameters permitting transitions between order and disorder. Non-classical statistics cannot make that distinction, by definition!!

Yet the quantum Hall dichotomy hurdle has persisted three decades, despite the fact that a joint description [9] had been reported shortly before the official discovery of the fractional effect at the Bell Telephone Laboratories in 1982. It may have been BTL's insistence that led a Nobel committee to de facto granting an award to the same effect twice, instead of awarding the act of injecting flux quanta [5.6] as alternative to charge quanta.

The quantum Hall literature has now become a living testimony to the aftermath of faulty Copenhagen's Doctrine. An examination of the 30 year review z[12] goes through the whole gamut from Schrödinger, many body alternatives, fractional charge, and for the purpose designed particles as well as injecting a delayed topology way down the road; instead of looking at the beginning where omission originated. These ideas kept surrounding the quantum Hall dichotomy to finally homing in back on flux quantization of the Sixties and charge quantization[3], injected as known results, not mentioning the sources [4,5,6] that brought them to the fore as primary pre-statistic sources.

The solid-state branch has been known for its very own views of physical reality. This new sub-branch dealing with the fractional quantum Hall effect has been acting as if their life in physics was solely activated by the wonderful new revelations of the Eighties. Let it be known people outside that branch also marvel at other aspects of these findings.

Reading the text next to Fig.2 of [12] is to me a de facto recognition of flux quantization [5, 6] as well as a recognition of charge quantization. The latter is really Gauss' law of electrostatics as charge counter, old but still true. Here it counts electrons jointly circulating at cyclotron rate, meaning they remain enclosed by a not simply connected hollow ring of integration. It means calling on a space-time version of Gauss' theorem as necessary when this theorem is used in the context of de Rham theory.

The fact is flux- together with charge quantization, without new artifacts, are joint starting points for a unified description covering integer and fractional filling factors of the QH effect. Yet, a vast majority cannot accept premises established in [4,5,6]? This insistence has led to an ensuing dichotomy that tears apart the coherence and basic logical consistency of contemporary physics! Confessing to a good faith misjudgment is never shameful; all us have been misled for many decades!

The Aharonov-Bohm integral 1959 and the space-time extended Gauss-Ampère law 1835 have been around for a long time. The one and only major point of obstruction I can see points at Copenhagen's Doctrine. It is an uncomfortable thought to see the existence of pre-metric and pre-statistic quantum relations as somehow washed out in an ever-present and everlasting statistic quantum background. Low temperature technology has shown that nature has been kinder to us than that. Here is the reason why people starting out with a statistics-based Schrödinger approach keep running into the roadblock of fractional quantization. It is the reluctance of admitting how a pre-statistic two quantum number global quantization of the Hall impedance experimentally shows Copenhagen's Doctrine in error.

While Schrödinger's equation and its one quantum number retained sole rule in this vast realm of physics, elsewhere authors of [4,5,6] have been pioneering new territory. An ironic side effect of Copenhagen Doctrine was a near-rejection of earlier recipe conditions as merely approximate. It marginalized Bohr's historic condition later revived by the Aharonov-Bohm integral; compare hereto the end of section 5.

Finally there is the choice between options I and II. Fig.1 based on option II produces numbers {2,3,4,5,6,7,8} whereas the choice h/4e gives a sequence alternative {1/2, 3/4, 1, 5/4, 3/2, 7/4, 2}, all of which shows how an integer-fractional distinction is still in the eye of the beholder. Yet an examination of the less than perfect plateaus and resistance dips alternating with good plateaus and zero dips hints at a conceivable role of Cooper-pairing. Independent n and s information will be crucial though for definitive n and s values.

While the pre-statistic ordered global process as applied to the quantum Hall effect opens new perspectives on ordered many particle phenomena, by the same token it raises questions about Schrödinger's many particle procedure. These are all ramifications inviting changes to be considered on the road towards welcoming a friendlier and more social-incorporating quantum realm into the general body of physical knowledge.

Note how the traditional method of solving differential equations works from the inside outwards by subsuming local validity of the equation and adjusting solutions to outside boundaries. By contrast, period integrals with a verified quantum message

[3] see text of ref.12 , and the explanation of filling factors to the right of its Fig.2.

have a potential of probing inwards from the outside, aiming at topology response directives from the inside. This inward probing seems unrestricted by microscopic metric considerations.

So far, this exploration ironically confirms an extended measure of Einstein's covariant description to include metric-free topological invariance for the quantum realm. Since the latter no longer permits a raising and lowering of indices as in standard tensor methods, transformation specifics now hold a more critical role. Taking e and h as a priori topological invariants, the dimensional basis $\{h,e,l,t\}$ becomes a big help in establishing pre-metric tensor species eligible in defining a specially invariant class of differential forms.

While nonclassical statistics was a steep eighty-year hurdle to identify as the trouble of Copenhagen's 1927 response, credit goes to Covariance making it look leaner and cleaner. Had Popper reinstated classical statistics in 1935, the 80-year conceptual hurdle might have resolved itself in 8 years. Planck inadvertently sounded the earliest ensemble warning, a decade and a half prior to the Schrödinger event [9], yet the majority did not budge. Popper almost succeeded, but sadly stopped short reinstating normal statistics.

This overview of an entangled nonclassical web invites a much-needed reconsideration of basics. Brief and simple, Copenhagen went overboard calling on nonclassical prerogatives. The fact is: a saner statistical proposition for Popper's ensemble is truly closer to physical reality than Copenhagen's Doctrine with its nonclassical aberrations.

References

[1] R. Courant, D. Hilbert, **Methods of Mathematical Physics**, German text (Springer, 1924).

[2] Karl R. Popper, **The Logic of Scientific Discovery** (New York, 1959).

[3] C. J. Davisson, L. H. Germer, *Phys. Rev.* **30**: 705 (1927); W. H. Bragg, W. L. Bragg, "X-ray Precursor" *Proc. Roy. Soc. London* **88**: 428 (1913).

[4] Y. Aharonov, D. Bohm, *Phys. Rev.* **115**: 485 (1959).

[5] R. Doll, M. Näbauer, *Phys. Rev. Lett.* **7**: 51 (1961).

[6] B. S. Deaver, W. M. Fairbank, *Phys. Rev. Lett.* **7**: 43 (1961).

[7] B. Josephson, *Phys. Lett.* **1**: 251 (1962).

[8] R. M. Kiehn, *J. Math. Phys.* **18**: 614 (1977).

[9] E. J. Post, **Quantum Reprogramming** (Kluwer 1995, Springer 2005); compare "Quantum Hall Effect as Ratio of Flux and Charge Quanta", *Found. of Phys.* **12** (2): 169 p.194 (1982); "Can Microphysical Structure be Probed by Period Integrals?", *Phys. Rev. D* **25** (12): 3223-3229 (1982).

[10] E. Cartan, *Ann. École Norm. Sup.* **41**: 1-12 (1924).

[11] G. de Rham, **Variétés Differentiables** (Paris,1955)

[12] Tapash Chakraborty, Klaus von Klitzing , **Thirty Years of Quantum Hall Effect** (Seminaire Poincaré, Feb 2011).

[13] E. J. Post, **Formal Structure of Electromagnetics** (Dover 1997)

[14] J. A. Schouten, **Tensor Analysis for Physicists** (Dover, 1989).