$E = mc^2$ and Einstein's Failure

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The famous formula $E = mc^2$ is a direct consequence of Maxwell´s theory of electromagnetic radiation, known long before its mistaken derivation by A. Einstein from the relativistic viewpoint.

1. Analysis

According to Maxwell´s theory of electromagnetic radiation the momentum carried by electromagnetic energy of a quantity E is E/c .

Let E is the energy of emitted photon (quantum of electromagnetic radiation). The question is: „What is the amount of mass m released of a body by emission of a photon, so what mass m is transmitted by a photon with its energy E?" The photon momentum is $p = E/c$ according to the Maxwell theory. As this photon carries the emitted mass m by the speed c, so the momentum of transmitted mass is mc, which corresponds to the photon momentum E/c , where:

$$\frac{E}{c} = mc \ ,$$

$$E = mc^2 \ .$$

The released (lost) mass m corresponds to the emitted energy mc^2 . It is evident that the photon is a basic structural unit of matter existing in a form of free energy or bind energy in material particles and interactions. It is strange, that physicists do not try to search for the essence of a photon and so discover the basic structural unit of mater and energy. In my monograph „God and the Universe" [1] the bipolar structure of a photon is disclosed causing the photon pulsation (vibration) and being the basic building block of mater – energy – space.

The above short analysis proves that Maxwell is a true father of the relation of mass-energy equivalence. This relation was known for his contemporaries and many of scientists were fascinated by the fact, that a huge amount of energy can be released from a small mass. This relation was presented and published in various versions by scientists, as mentioned in Wikipedia, like S. Tolver Preston (1875), J. Thomson (1881), Oliver Heaviside (1888), George Searle (1897), Wilhelm Wien (1900), Max Abraham (1902), Hendrik Lorentz (1904), F. Hasenöhrl (1904). Henri Poincaré (1900) used the expression $m = E/c^2$ for the mass of electromagnetic energy. Olinto de Pretto (1903) presented a mass–energy relation in the exact form. De Pretto's paper received recent press coverage when Umberto Bartocci came to the conclusion that Einstein was probably aware of De Pretto's work.

As Einstein was also fascinated by this relation known already in his period, he tried to derive it from the relativistic viewpoint, but he failed as shown by the following analysis.

Einstein presented the derivation of the famous formula in his article „Does the Inertia of a Body Depend Upon its Energy Content?" published September 27, 1905 in the *Annalen der Physik* [1].

2. Einstein´s Derivation

Let a system of plane waves of light, referred to the system of co-ordinates possess the energy L; let the direction of the ray makes an angle φ with the axis of x of the system. If we introduce a new system of coordinates moving in uniform parallel translation with respect to the system at rest along the axis of x with the velocity v, then this quantity of light measured in the moving system possesses the energy

$$L^* = L\frac{1-\frac{v}{c}\cos\varphi}{\sqrt{1-v^2/c^2}} \ ,$$

where c denotes the velocity of light.

Let there be a stationary body and let its energy referred to the system at rest be E_0 . Let the energy of the body relative to the system moving with the velocity v, be H_0 .

Let this body send out, in a direction making an angle φ with the axis of x, plane waves of light, of energy ½L measured relatively to the system at rest and simultaneously an equal quantity of light in the opposite direction. Meanwhile the body remains at rest with respect to the system at rest. If we call the energy of the body after the emission of light E_1 or H_1 respectively, measured relatively to the system at rest or the moving system respectively, then by employing the relation given above we obtain:

$$E_0 = E_1 + \tfrac{1}{2}L + \tfrac{1}{2}L \qquad (1)$$

$$H_0 = H_1 + \tfrac{1}{2}L\frac{1-\frac{v}{c}\cos\varphi}{\sqrt{1-v^2/c^2}} + \tfrac{1}{2}L\frac{1+\frac{v}{c}\cos\varphi}{\sqrt{1-v^2/c^2}} = H_1 + \frac{L}{\sqrt{1-v^2/c^2}} \qquad (2)$$

As $H - E$ can only differ from the kinetic energy K by an additive constant, the difference between kinetic energies of a body with respect to the moving system before and after emission of light is:

$$K_0 - K_1 = H_0 - E_0 - (H_1 - E_1) = L\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) . \qquad (3)$$

The kinetic energy of the body with respect to moving system diminishes as a result of the emission of light. Neglecting magnitudes of fourth and higher orders we may place

$$K_0 - K_1 \cong \frac{1}{2}\frac{L}{c^2}v^2 \ . \qquad (4)$$

From this equation it directly follows: "*If a body gives off the energy L in the form of radiation, its mass diminishes by* L/c^2 ." The fact that the energy withdrawn from the body becomes energy of radiation evidently makes no difference, so that we are led to the more general conclusion that *the mass of a body is a measure of its energy-content*.

3. Comments to Einstein´s Invalid Derivation

Let us look at the error in Einstein´s derivation and conclusion. Eq. (2) means that the emitted energy increases from the value L to the value L/β with respect to the system moving with a uniform velocity v, where $\beta \equiv \sqrt{1-v^2/c^2}$. According to Einstein, the increase of emitted energy of a body $\Delta L = L/\beta - L = L\left(\beta^{-1}-1\right)$ means at the same time the decrease of its kinetic energy with respect to moving system is

$$K_0 - K_1 = L\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right) = L\left(\beta^{-1}-1\right) = \Delta L \quad . \tag{6}$$

As the velocity v does not change, the correct conclusion is that this additional loss of kinetic energy $\Delta mv^2/2$ can only be the consequence of the relativistic mass decrease Δm of the body with respect to the moving system, where

$$K_0 - K_1 = \frac{\Delta mv^2}{2} \quad . \tag{5}$$

By accepting the formula $E = mc^2$, we have

$$\Delta m = \frac{L}{c^2}\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right) \quad , \tag{6}$$

but according to Einstein´s mistaken conclusion it is

$$\Delta m = \frac{L}{c^2} \quad . \tag{7}$$

The difference between the correct (6) and erroneous (7) formulas is principal. The relativistic loss of mass Δm corresponds only to the decrease of kinetic energy of a body corresponding to the additional increase of emitted energy ΔL towards the moving system, but not to the whole emitted energy L as Einstein erroneously concluded. The difference between the Eqs. (6) and (7) is evident: if the speed of a moving system is $v = 0$ (system is at rest towards the emitting body), the correct result is $\Delta m = 0$, but not $\Delta m = L/c^2$ as Einstein received.

Einstein´s declaration, that the difference $K_0 - K_1$ is a result of the whole emitted energy L, is a crucial moment of his failure leading to invalid deduction, that mass diminishes by L/c^2, which have no relation to his really derived facts. He could only compare the loss of relativistic mass Δm with the increased energy ΔL thanks to relative velocity v of moving system with respect to the emitting body. By this correct approach, Einstein could never derive the famous formula.

In order to obtain the famous formula, Einstein transformed the relation $\Delta L = L\left(\beta^{-1}-1\right)$ to the relation $\Delta L = Lv^2/2c^2$ by the procedure of neglecting magnitudes of fourth and higher orders, which is inadmissible for the relativistic conditions, as the difference $\Delta L = L\left(\beta^{-1}-1\right)$ can be remarkable only by high speeds v close to the speed of light **c** and, at these speeds, the correct formula $\Delta L = L\left(\beta^{-1}-1\right)$ gives quite different results than Einstein´s wrong formula $\Delta L = Lv^2/2c^2$.

Example: The excited proton with a velocity of $v = 0.9c$ emits two photons of total energy L in two opposite directions, so that the emitted energy increases from L to L/β .

According to the correctly derived formula, the increase of emitted energy of a proton, as well as the decrease of its kinetic energy, is: $\Delta L = K_0 - K_1 = 1.294L$. According to the wrong modified formula it is $\Delta L = 0.405L$. The correct result is three times bigger than the wrong one. If the speed of a proton is $v = 0.99c$, the correct formula gives the result $\Delta L = 6.09L$, while the wrong one: $\Delta L = 0.49L$, so the correct result is twelve times bigger than the wrong one.

Saying that the formulas $\Delta L = L\left(\beta^{-1}-1\right)$ and $\Delta L = Lv^2/2c^2$ are equivalent at the small speeds, both giving results close to zero, is a fatal mistake, as the formula $\Delta L = L\left(\beta^{-1}-1\right)$ is not received for low speeds of classical physics, but just for relativistic effects, what means, that using the deformed formula $\Delta L = Lv^2/2c^2$ for relativistic purposes gives totally wrong results and so is unacceptable.

The above results deny the correctness of Einstein´s derivation of the famous formula $E = mc^2$.

4. Conclusion

The formula $E = mc^2$ is classical and non-relativistic. Einstein´s conclusion about mass-energy relation has no validity because of its invalid derivation, what disqualifies him as originator of the famous formula.

References

[1] P. Kohut, **The Nature of the Universe** (VDM Verlag Dr. Muller, Saarbrucken, 2011).

[2] A. Einstein, "Ist die Trägheit eines Körpers von seinem Energiegehalt abhängig?" *Annalen der Physik* **17**: 891 (1905).