

A reinterpretation of the von Klitzing and Josephson constants

March 4, 2011

Abstract

A consideration of the von Klitzing and Josephson constants is made, showing what their values would be in a system with Planck sized charges and reinterpreting the relationships uncovered. The reinterpretation is based on a dimensional analysis that concludes with the representation of many Planck and fundamental parameters in terms of ratios of only R_k and K_j or only h and c and may enable advances in metrology using the more precisely known values of R_k and K_j .

Author: M Lawrence

Maldwyn Theoretical Physics

mike.lawrence@maldwynphysics.org

PACS: 13.40.Em, 06.20.Jr, 06.30.Ka, 14.60.Fg, 12.20.Ds, 12.10.-g

A theoretical underpinning of the von Klitzing and Josephson constants

This paper will show that by using the appropriate units for fundamental parameters, the von Klitzing and Josephson constants drop out as the maximal values of resistance and twice the inverse of magnetic flux respectively. The fundamental parameters will be displayed in SI values for their new units and the relationships between those parameters made clearer than has been hitherto possible. The end result is the display of many parameters in terms of only h , c and $\sqrt{\alpha/2\pi}$, although there is the occasional use of $\sqrt{10^7}$ due to current SI misalignment between electromagnetic and mechanical parameter units.

The starting units used here, to which the new units will be compared, are based on the properties of Planck mass size M_o and Planck charge size Q_o respectively, where $M_o = \sqrt{hc/G} = Q_o c / \sqrt{G} \times 10^7$ rather than the more usual use of \hbar in producing a smaller Planck-related mass M_p . Also affected is the Planck length, where $L_o = h / (M_o c)$ rather than the more usual $L_p = \hbar / (M_p c)$. M_o , Q_o and L_o are part of what is termed here the Adjusted Planck Unit (APU) system. This starting system is used in order to eliminate the 2π factor that would otherwise clutter the consideration.

The hypothesized new unit system, dubbed Double Adjusted Planck units (DAPU) is based on a new DAPU mass M_* where $M_* = M_o \sqrt{G}$ and instead of the APU Planck length L_o , the DAPU length $L_* = L_o / \sqrt{G}$ is defined. The values of c and h remain unchanged, but there are knock-on effects for other mass-related parameters. To align the charge and mass side requires that the base unit sizes of the electron and Planck charges are decreased by the factor $\sqrt{10^{-7}}$ and the permeability u_o is defined to be equal to \sqrt{G} rather than the usual $4\pi \times 10^{-7}$. It is useful for display purposes to define a factor $d = \sqrt{\alpha/2\pi}$ that represents the ratio $d = q_{e*} / Q_*$, where Q_* is the DAPU unit of charge $Q_* = M_* / c$ and q_{e*} is the DAPU size of the electronic charge. The DAPU relationship between mass M_* and charge Q_* is different to that between the Planck mass M_p and Planck charge Q_p , where $Q_p = M_p \sqrt{2\pi G} \times 10^7 / c$ with the $\sqrt{10^7}$ factor being the misalignment factor previously mentioned.

The merging of G with the APU mass M_o to produce the DAPU mass M_* would seem to ignore the units of G , effectively treating G as unity and without units. But this is not the case. The units of G are $m^3 kg^{-1} s^{-2}$. A consideration of the standard laws of nature and the fundamental constants through dimensional analysis shows that

if each parameter is assigned an appropriate dimensionality, every fundamental constant, other than c , will have a total dimensionality of zero. The dimensionalities of the main SI, APU or DAPU parameters in terms of hypothetical dimension Y that emerge from the consideration are:

$$\text{Mass } M_* = Y^{+1} \text{ Velocity } c = Y^{+2} \text{ Length } L_* = Y^{-3} \text{ Charge } Q_* = Y^{-1} \text{ Time } T_* = Y^{-5} \text{ Energy } E_* = Y^{+5}$$

and of course

$$h = Y^0 \text{ and } G = Y^0$$

The units of G are $m^3kg^{-1}s^{-2} = Y^{-9}Y^{-1}Y^{+10} = Y^0$ dimensionality and h has units $kgm^2s^{-1} = Y^{+1}Y^{-6}Y^{+5} = Y^0$ dimensionality.

So the units of h and G are irrelevant because they represent a fundamental constant with zero dimensionality. Thus adjusting the APU mass to the DAPU mass involves only multiplying by a dimensionless number, and does not affect the dimensionality of the units of mass, other than changing the size of the base Planck mass unit.

Table 1 provides a list of the main parameters and their SI values at their Planck sizes. The column headed 'SI/new SI units' means that where the current electromagnetic SI units appear they have been adjusted by the factor $\sqrt{10^7}$ mentioned earlier and their use is denoted by a cedilla above the unit, thus $X(SI) \iff \tilde{X}(NSI)$. Note that the factor d does not appear in Table 1 because these values are all based on the DAPU charge Q_* .

Table 1: Maximal parameter values in SI units with electronic charge size Q_*

Parameter (X_*)	Planck DAPU unit's SI Value	SI/new SI Units	DAPU equivalent	As Constants
Gravitational Constant(G)	<i>none</i>	$m^3kg^{-1}s^{-2}$	<i>none</i>	<i>none</i>
Permeability(u_*)	$\sqrt{6.67428 \times 10^{-11}}$	NA^{-2}	<i>none</i>	\sqrt{G}
Angular Momentum(h)	$6.62606896 \times 10^{-34}$	Js	kgm^2s^{-1}	h
Mass(m_*)	$4.45695580009 \times 10^{-13}$	kg	kg	\sqrt{hc}
Magnetic Flux(ϕ_*)	$4.45695580009 \times 10^{-13}$	\tilde{W}	\sqrt{kgmms}^{-1}	\sqrt{hc}
Charge-mass(q_*c)	$4.45695580009 \times 10^{-13}$	$\tilde{C}ms^{-1}$	\sqrt{kgmms}^{-1}	\sqrt{hc}
Velocity(v_*)	2.99792458×10^8	ms^{-1}	ms^{-1}	c
Resistance(R_*)	2.99792458×10^8	$\tilde{\Omega}$	ms^{-1}	c
Momentum(m_*v_*)	$1.33616173451 \times 10^{-4}$	$kgms^{-1}$	$kgms^{-1}$	$c\sqrt{hc}$
Current(i_*)	$8.98755178737 \times 10^{16}$	\tilde{A}	\sqrt{kgms}^{-1}	c^2
Action(m_*/r_*)	$8.98755178737 \times 10^{16}$	kgm^{-1}	kgm^{-1}	c^2
Angular Frequency(w_*)	$6.04538245967 \times 10^{37}$	Hz	s^{-1}	$c^2\sqrt{c/h}$
Frequency(f_*)	$6.04538245967 \times 10^{37}$	Hz	s^{-1}	$c^2\sqrt{c/h}$
Energy(E_*)	$4.00571210673 \times 10^4$	J	kgm^2s^{-2}	$c^2\sqrt{hc}$
Potential Difference(v_*)	$2.69440024174 \times 10^{25}$	\tilde{V}	\sqrt{kgmms}^{-2}	c^3
Acceleration(a_*)	$1.81236006713 \times 10^{46}$	ms^{-2}	ms^{-2}	$c^3\sqrt{c/h}$
Magnetic Inductance(B_*)	$1.81236006713 \times 10^{46}$	$\tilde{A}m^{-1}$	ms^{-2}	$c^3\sqrt{c/h}$
Force(F_*)	$8.07760871306 \times 10^{33}$	N	$kgms^{-2}$	c^4
Electric Field(ξ_*)	$5.43331879307 \times 10^{54}$	$\tilde{V}m^{-1}$	$\sqrt{kgmm}^{-2}s^{-2}$	$c^4\sqrt{c/h}$
Mass Density(ρ_*)	$3.65466490836 \times 10^{75}$	kgm^{-3}	kgm^{-3}	c^5/h
Current Density(J_*)	$3.65466490836 \times 10^{75}$	$\tilde{A}m^{-2}$	$\sqrt{kgmm}^{-2}s^{-1}$	c^5/h
Power(P_*)	$2.42160617085 \times 10^{42}$	Js^{-1}	kgm^2s^{-3}	c^5
Pressure(p_*)	$3.28464901294 \times 10^{92}$	Nm^{-2}	$kgm^{-1}s^{-2}$	c^7/h
Energy Density(ϵ_*)	$3.28464901294 \times 10^{92}$	Jm^{-3}	$kgm^{-1}s^{-2}$	c^7/h
Charge(q_*)	$1.4866804288 \times 10^{-21}$	\tilde{C}	\sqrt{kgm}	$\sqrt{h/c}$
Conductance(ζ_*)	$3.33564095198 \times 10^{-9}$	$\tilde{\Omega}^{-1}$	$m^{-1}s$	c^{-1}
Moment(m_*r_*)	$2.21021869736 \times 10^{-42}$	kgm	kgm	h/c

Distance(L_*)	$4.9590321208 \times 10^{-30}$	m	m	$c^{-1}\sqrt{h/c}$
Inductance(\mathcal{L}_*)	$4.9590321208 \times 10^{-30}$	\tilde{H}	$\sqrt{kgmm}^{-1}s^{-1}$	$c^{-1}\sqrt{h/c}$
Permittivity(ε_*)	$1.36193500681 \times 10^{-12}$	Fm^{-1}	$m^{-2}s^2$	c^{-2}/\sqrt{G}
Time(T_*)	$1.65415506243 \times 10^{-38}$	s	s	$c^{-2}\sqrt{h/c}$
Area(A_*)	$2.45919995751 \times 10^{-59}$	m^2	m^2	h/c^3
Volume(V_*)	$1.21952515808 \times 10^{-88}$	m^3	m^3	$h\sqrt{h/c}/c^4$

In DAPU the value of each parameter in Table 1 is 1. To arrive at the maximal real values that can be found experimentally, the list needs to be adjusted to use q_e instead of Q_* since we do not observe Q_* charges usually. The maximal values in SI units of some parameters under this limitation are listed in Table 2. Note that the factor d is inversely proportional to the dimensionality of every parameter

Table 2: Maximal parameter values in SI units using electronic charge size q_{e*}

Parameter (X_{e*})	Maximal DAPU unit's SI Value	SI/new SI Units	DAPU equivalent	As Constants
Permeability(u_{e*})	$\sqrt{6.67428 \times 10^{-11}}$	NA^{-2}	<i>none</i>	\sqrt{G}
Angular Momentum(h)	$6.62606896 \times 10^{-34}$	Js	kgm^2s^{-1}	h
Mass(m_{e*})	$1.30781284183 \times 10^{-11}$	kg	kg	$d^{-1}\sqrt{hc}$
Magnetic Flux(ϕ_{e*})	$1.30781284183 \times 10^{-11}$	W	\sqrt{kgmms}^{-1}	$d^{-1}\sqrt{hc}$
Charge-mass($q_{e*}c$)	$1.30781284183 \times 10^{-11}$	Cms^{-1}	\sqrt{kgmms}^{-1}	$d^{-1}\sqrt{hc}$
Velocity(v_{e*})	$2.58128075573 \times 10^{11}$	ms^{-1}	ms^{-1}	$d^{-2}c$
Resistance(R_{e*})	$2.58128075573 \times 10^{11}$	Ω	ms^{-1}	$d^{-2}c$
Momentum($m_{e*}v_{e*}$)	$3.37583212072 \times 10^{00}$	$kgms^{-1}$	$kgms^{-1}$	$d^{-3}c\sqrt{hc}$
Current(ι_{e*})	$6.66301033990 \times 10^{22}$	A	\sqrt{kgms}^{-1}	$d^{-4}c^2$
Action(m_{e*}/r_{e*})	$6.66301033990 \times 10^{22}$	kgm^{-1}	kgm^{-1}	$d^{-4}c^2$
Angular Frequency(w_{e*})	$1.31510410477 \times 10^{45}$	Hz	s^{-1}	$d^{-5}c^2\sqrt{c/h}$
Frequency(f_{e*})	$1.31510410477 \times 10^{45}$	Hz	s^{-1}	$d^{-5}c^2\sqrt{c/h}$
Energy(E_{e*})	$8.71397048778 \times 10^{11}$	J	kgm^2s^{-2}	$d^{-5}c^2\sqrt{hc}$
Potential Difference(v_{e*})	$1.71991003656 \times 10^{34}$	V	\sqrt{kgmms}^{-2}	$d^{-6}c^3$
Acceleration(a_{e*})	$3.39465291742 \times 10^{56}$	ms^{-2}	ms^{-2}	$d^{-7}c^3\sqrt{c/h}$
Magnetic Inductance(B_{e*})	$3.39465291742 \times 10^{56}$	Am^{-1}	ms^{-2}	$d^{-7}c^3\sqrt{c/h}$
Force(F_{e*})	$4.43957067897 \times 10^{45}$	N	$kgms^{-2}$	$d^{-8}c^4$
Electric Field(ξ_{e*})	$8.76255224813 \times 10^{67}$	Vm^{-1}	$\sqrt{kgmm}^{-2}s^{-2}$	$d^{-9}c^4\sqrt{c/h}$
Mass Density(ρ_*)	$1.72949880638 \times 10^{90}$	kgm^{-3}	kgm^{-3}	$d^{-10}c^5/h$
Current Density(J_{e*})	$1.72949880638 \times 10^{90}$	Am^{-2}	$\sqrt{kgmm}^{-2}s^{-1}$	$d^{-10}c^5/h$
Power(P_{e*})	$1.14597783573 \times 10^{57}$	Js^{-1}	kgm^2s^{-3}	$d^{-10}c^5$
Pressure(p_{e*})	$1.15236684298 \times 10^{113}$	Nm^{-2}	$kgm^{-1}s^{-2}$	$d^{-14}c^7/h$
Energy Density(ϵ_{e*})	$1.15236684298 \times 10^{113}$	Jm^{-3}	$kgm^{-1}s^{-2}$	$d^{-14}c^7/h$
Charge(q_{e*})	$5.06652691277 \times 10^{-23}$	C	\sqrt{kgm}	$d\sqrt{h/c}$
Conductance(ς_{e*})	$3.87404585022 \times 10^{-12}$	Ω^{-1}	$m^{-1}s$	d^2c^{-1}
Moment($m_{e*}r_{e*}$)	$2.56696949578 \times 10^{-45}$	kgm	kgm	d^2h/c
Distance(L_{e*})	$1.96279575614 \times 10^{-34}$	m	m	$d^3c^{-1}\sqrt{h/c}$
Inductance(\mathcal{L}_{e*})	$1.96279575614 \times 10^{-34}$	H	$\sqrt{kgmm}^{-1}s^{-1}$	$d^3c^{-1}\sqrt{h/c}$
Permittivity(ε_{e*})	$1.83707675365 \times 10^{-18}$	Fm^{-1}	$m^{-2}s^2$	d^4c^{-2}/\sqrt{G}
Time(T_{e*})	$7.60396075393 \times 10^{-46}$	s	s	$d^5c^{-2}\sqrt{h/c}$
Area(A_{e*})	$3.85256718034 \times 10^{-68}$	m^2	m^2	d^6h/c^3
Volume(V_{e*})	$7.56180251183 \times 10^{-102}$	m^3	m^3	$d^9h\sqrt{h/c}/c^4$
Capacitance(C_{e*})	$2.94580926040 \times 10^{-57}$	F	$m^{-1}s^2$	$d^7c^{-3}\sqrt{h/c}$

The important point is that the maximal value for Resistance R_{e^*} is the von Klitzing constant R_k , $R_{e^*} = R_k$ and the value of the Magnetic Flux ϕ_{e^*} is twice the inverse of the Josephson constant K_j , $\phi_{e^*} = (2/K_j)$. Table 3 shows that the SI values of R_k and K_j are identical to R_{e^*} and $2/\phi_{e^*}$ when translated into DAPU units. To ensure clarity, new parameters will be defined $R''_k = R_{e^*} = R_k$ and $K''_j = 2/\phi_{e^*} = K_j$ where R''_k and K''_j denote the DAPU interpretations of R_k and K_j which can be more easily compared with other constants in DAPU units. Also note that each parameter in Table 2, apart from the permeability and permittivity which have been defined to be related to G , can be described in terms of only h , c and in some cases the factor d .

Some of the parameters, such as velocity v_{e^*} , appear to be larger than their Planck versions. As will be shown below, it is possible to interpret R_k as equivalent to a velocity and, if so, this suggests that faster than light travel through media may be a possibility and should be investigated. This result, where the parameters $X_{e^*} > X_*$, will be considered further later.

All these parameters above have been produced using standard relationships and formulae. It is interesting to observe that some parameters on the mass side have identical sized partners on the charge size, for example mass M_* and Magnetic Flux ϕ_* . One interpretation could be that magnetic flux is the equivalent of the mass in an electromagnetic system, and that resistance R_{e^*} is the equivalent of velocity v_{e^*} . Dimensional analysis supports this and the appropriateness of this interpretation is shown below.

To ensure that the above values can be understood properly, the following series of relationships at the Q_* level can be culled from the standard laws and the results computed and confirmed to be correct using their SI values in Table 1 as:

$$F_* = (M_*/L_*)^2 = (\phi_*/L_*)^2 = (Q_*c/L_*)^2 = M_*a_* = \phi_*B_* = Q_*cB_* = Q_*\xi_* = V_*c = i_*^2 = hc/L_*^2$$

It is also possible to use the same relationships at the q_{e^*} level, using the parameter values from Table 2 thus:

$$F_{e^*} = (M_{e^*}/L_{e^*})^2 = (\phi_{e^*}/L_{e^*})^2 = (Q_{e^*}c/L_{e^*})^2 = M_{e^*}a_{e^*} = \phi_{e^*}B_{e^*} = Q_{e^*}cB_{e^*} = Q_{e^*}\xi_{e^*} = V_{e^*}c = i_{e^*}^2 = hc/L_{e^*}^2$$

Since the values of some electromagnetic parameters are identical to the values of some mechanical parameters, it suggests that mechanical formulae could be used with electromagnetic parameters substituted instead, and vice versa. One example would be the simple $L_{e^*} = v_{e^*}T = \mathcal{L}_{e^*}$ which suggests that in some way electromagnetic inductance is equivalent to a mechanical distance.

Now it is possible to reinterpret the only two fundamental constants left, aside from the factor d which defines the electron charge-based system that we experience because of the relative size of the charge on the electron q_{e^*} versus the DAPU Planck charge Q_* , in term of the two base parameters which have only dimension $Y^{\pm 1}$, that is charge Q_* and mass M_* .

$$M_*Q_* = h$$

$$M_*/Q_* = c$$

So the two constants h and c represent the only two possible ratios of the DAPU mass and DAPU charge, each used once. That ought to infer something fundamental about the underlying structure of matter.

Also important is that the same reinterpretation can be done for h and c using R_k and K_j . However, for consistency, the DAPU constants R''_k and K''_j will be used, but the same relationships remain.

$$R''_k(K''_j/2)^2 = h$$

$$R''_k = c/d^2$$

The comparison with M_* and Q_* is not identical because, although K''_j is an inverse magnetic flux and so equivalent to an inverse mass of size $2/K''_j$, R''_k is not a charge, but a resistance or velocity. But by using the same relationship between equivalent mass and charge, with a factor of 2, the value of h can be recovered as:

$$\text{Equivalent mass x electron charge} = (2/K''_j)q_{e^*} = h$$

And the same can be achieved with the ratio of equivalent mass to charge to recover the velocity c/d^2 or R''_k as:

$$\text{Equivalent mass / electron charge} = (2/K''_j)/q_{e*} = h/q_{e*}^2 = h/[d^2Q_*^2] = Q_*^2c/[d^2Q_*^2] = c/d^2 = R''_k$$

This shows that if inverse magnetic flux can be considered as equivalent to a mass, then resistance can be considered as equivalent to a velocity.

It is possible to generate examples of the usual constants of nature or DAPU parameters, other than G which was factored into M_* , using just R''_k and $K''_j/2$ as follows:

$$\begin{aligned} M_* &= d/(K''_j/2) \\ M_{e*} &= 1/(K''_j/2) = \phi_{e*} \\ q_{e*} &= 1/[R''_k(K''_j/2)] \\ Q_* &= 1/[R''_k(K''_j/2)d] \\ L_* &= 1/[R''_k^2(K''_j/2)d^3] \\ L_{e*} &= 1/[R''_k^2(K''_j/2)] \\ T_* &= 1/[R''_k^3(K''_j/2)d^5] \\ h &= 1/[R''_k(K''_j/2)^2] \\ E_* &= R''_k^2/[(K''_j/2)d^5] \\ c &= R''_kd^2 \\ \iota_{e*} &= R''_k^2 \\ \vee_{e*} &= R''_k^3 \end{aligned}$$

These relationships can be checked by using the following standard law formulae, in either Q_{e*} or q_{e*} form, and the DAPU values of the parameters in Table 1 or 2:

$$\vee_{e*} = \iota_{e*} \times R''_k \text{ or } B_{e*} = \phi_{e*}/A_{e*} \text{ or } E_{e*} = m_{e*} \times v_{e*}^2 \text{ or } F_{e*} = p_{e*} \times A_{e*} \text{ or } q_{e*} = \iota_{e*} \times T_{e*} \text{ or } E_{e*} \times T_{e*} = h$$

Many parameters have been left out of the list for brevity, including those based on materials which require the permeability factor u_{e*} or u_* and would mean the inclusion of \sqrt{G} in the formula of constants producing those parameters.

Table 3 shows the relative factors required to translate between DAPU/APU/SI units. The SI values should be multiplied by the the factors in the appropriate column to produce the DAPU or APU values of that parameter.

Table 3: Translating between units

Parameter	DAPU factor X_*	APU factor X_o	SI value of Planck unit	SI Name
h	2π	2π	$1.0545716 \times 10^{-34}$	\hbar
M	$\sqrt{2\pi G}$	$\sqrt{2\pi}$	2.1764374×10^{-8}	M_{Planck}
Q	$\sqrt{10^{-7}}$	$\sqrt{10^{-7}}$	$4.7012963 \times 10^{-18}$	Q_{Planck}
q_e	$\sqrt{10^{-7}}$	$\sqrt{10^{-7}}$	$1.6021765 \times 10^{-19}$	e
L	$\sqrt{2\pi/G}$	$\sqrt{2\pi}$	$1.6162525 \times 10^{-35}$	L_{Planck}
G	<i>none</i>	1	6.67428×10^{-11}	G
c	1	1	2.99792458×10^8	c
R_{e*}	10^{-7}	10^{-7}	2.581280756×10^4	R_k
$2/\phi_{e*}$	$\sqrt{10^{-7}}$	$\sqrt{10^{-7}}$	$4.835978909 \times 10^{14}$	K_j

It may be possible to improve the accuracy of measurement of some of the constants by using the new relationships uncovered between R''_k and K''_j . It is not only h that can be made more precisely from ratios of R_k and K_j . There are many more composites of R''_k and K''_j that produce other parameters which may not have been measured to as

great an accuracy as R_k and K_j have been. Unfortunately G will not be one such open to improvement unless its square root equivalence to permeability u_o , as defined in the DAPU system of units, is confirmed as appropriate, when it will become the accuracy of measurement of permittivity ε_o that will set the metrology limits for precision of the value of G .

Discussion

The parameters in Table 2, based on q_{e*} , that have $X_{e*} > X_*$, have sizes greater than their Planck DAPU values in Table 1. This leads to parameters like $v_{e*} = cd^{-2} = 2\pi c/\alpha$ which is greater than light speed. It is the d factor, the ratio q_{e*}/Q_* , that alters the parameter values in Table 2. Where the parameter has d^{+x} the parameter will be smaller than its Planck parent and where the parameter has d^{-x} it will be larger - the whole parameter space has been stretched out of symmetry, even though the same laws and relationships apply.

For all physical objects at the maximal values for each parameter possessed, the actual value of d is immaterial. Such objects obey the same laws regardless of the relative size of the electron charge q_{e*} to Planck charge Q_* . It is only at the lower levels, below maximal values, that the ratio of d to, for example, the masses of the loops will produce different physical effects, such as differing electron energy levels in atoms, dependent on the size of the electron loop.

Whether the maximal values $X_{e*} > X_*$ can actually be attained is a question for experimental verification or rebuttal. That R_k and K_j have been measured to be the sizes that they are [1] makes it certain that some of them can, since $R_k = c/d^2 = \sqrt{2\pi/\alpha c}$. So although equivalent to a velocity, it is not clear if $R_k > c$ means that q_{e*} based objects can exceed c in velocity. So there may be limits on the interpretation of 'equivalent to' when two DAPU parameters have identical values and dimensionalities. What is physically possible may depend on whether the parameter under consideration is from the mechanical or electromagnetic part of parameter space.

Conclusions

The reinterpretation of R_k and $K_j/2$ as velocity and inverse mass respectively, together with their current excellent precision of measurement, should enable increased accuracy in the estimation of the values of other parameters and fundamental constants identified as novel composite functions of R''_k and $K''_j/2$.

References:

[1] Mohr P J, Taylor B N, Newell D B "CODATA Recommended values of the fundamental physical constants: 2006" Rev. Mod. Phys. 80 633 (2008); Mohr P J, Taylor B N, Newell D B arXiv:0801.0028; <http://dx.doi.org/10.1103/RevModPhys.80.633>; <http://physics.nist.gov/cuu/Constants/index.html>