A Model of the Electron Based on Its Magnetic Moment

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The magnetic moment can be expressed as the product of a "moment of charge" and a frequency. Using this, the Bohr magneton appears to be the electron magnetic moment generated when the electron is modeled as a hollow sphere. It is shown that, by treating the electron as a small charged particle in a rotating high-speed orbit instead of a hollow sphere, a correction to the hollow-sphere moment of charge approximation appears that produces an electron magnetic moment much closer to the actual value than the Bohr magneton.

1. Introduction

In my Alpha-Beta model of the atomic nucleus, I model the electron as a beta particle orbiting an antineutrino [1]. While this model may seem arbitrary, it was actually formulated based on the electron's magnetic moment and its deviation from the Bohr magneton. The discussion that follows gives a brief overview of the reasoning that produced the model.

2. The Magnetic Moment

Long ago (around 1979) I noticed that the dimensions of the magnetic moment – usually expressed as Joules per Tesla (J/T) reduce down to Coulombs meters-squared per second (C-m²/s). This makes magnetic moment look like the product of a "moment of charge" and a frequency; with this moment of charge that I speak of being the charge-equivalent of the moment of inertia for mass. Therefore, the magnetic moment μ of a particle could be expressed as

$$\mu = I_q \nu, \tag{1}$$

where I_q is the moment of charge of the particle and ν is its rotating frequency. This meant that since the moment of inertia (mass) for a solid sphere is [2]

$$I = \tfrac{2}{5} m r^2, \tag{2}$$

and for a hollow sphere [2],

$$I = \tfrac{2}{3} m r^2; \tag{3}$$

the moment of charge for a solid sphere would be

$$I = \tfrac{2}{5} q r^2, \tag{4}$$

and for a hollow sphere,

$$I = \tfrac{2}{3} q r^2, \tag{5}$$

where m and q are mass and charge, respectively.

In the case of the electron, since its (classical) radius is roughly twice that of the proton, but its mass is 1,836 times smaller than the proton's; assuming the density of nuclear matter is constant; it appears to be a hollow particle. This makes its moment of charge

$$I_e = \tfrac{2}{3} q_e {r_e}^2. \tag{6}$$

The electron magnetic moment [3],

$$\mu_e = -9.28476x10^{-24}\, J/T, \tag{7}$$

is approximated by the Bohr magneton [4],

$$\mu_B = \frac{q_e \hbar}{2m_e} = -9.27401x10^{-24}\, J/T, \tag{8}$$

where $\hbar$ is Planck's constant, with

$$\mu_e = 1.00116 \mu_B. \tag{9}$$

Assuming (for the moment) that the Bohr magneton is the electron magnetic moment, the electron's rotating frequency becomes

$$\mu_e = \mu_B, \tag{10}$$

$$I_e \nu_e = \frac{q_e \hbar}{2m_e}, \tag{11}$$

$$\tfrac{2}{3} q_e {r_e}^2 \nu_e = \frac{q_e \hbar}{2m_e}, \tag{12}$$

$$\nu_e = \frac{3\hbar}{4 m_e {r_e}^2}. \tag{13}$$

Knowing, however, that the electron's magnetic moment is slightly larger than the Bohr magneton is what led to my model of the electron. I reasoned that, if the electron was actually a small charged particle in a high-speed orbit about some central entity, the progression of the orbit about an axis of revolution through the central entity would make the electron appear to be a hollow sphere. This is why the Bohr magneton so nearly matches the electron's magnetic moment. The moment of charge of a hollow sphere approximates the moment of charge of the electron's orbiting particle. But – it is not exactly the electron's moment of charge.

3. The Electron Model

In Fig. 1 below, I try to illustrate the model I am proposing. The small circle is the charged particle (what I now call a beta particle), the large circle is the orbit of the charged particle, and the vertical line dissecting the large circle is the axis about which the orbit is spinning. The electron appears to be spinning about this axis so the moment of charge of the electron is taken about this axis.

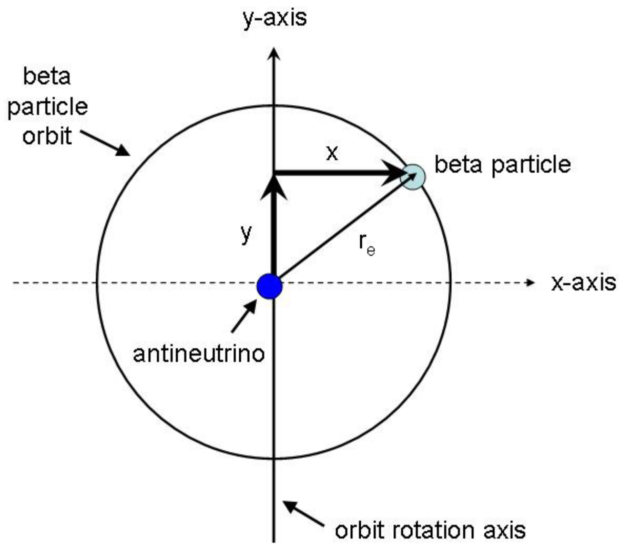

Fig. 1. Diagram of the electron as a beta particle orbiting an antineutrino. As the beta particle orbits, the orbit rotates about the *y*-axis creating the illusion of a hollow sphere.

This configuration gives the electron's moment of charge two components. Because the orbit of the particle is spinning on an axis, the parallel axis theorem [5] must be applied to calculate the charge moment. Briefly, the parallel axis theorem states that the moment of inertia of a body with respect to an axis not through the body is equal the moment of inertia of the body, plus the product of the mass of the body and the average distance squared the body is from the desired axis, or

$$I' = I + md^2. \tag{14}$$

Applying this concept to the moment of charge gives

$$I' = I + qd^2, \tag{15}$$

or in the case of the electron,

$$I_e = I_{bp} + q_{bp}d^2, \tag{16}$$

where the subscript *bp* denotes the beta particle and d^2 is the average square distance the beta particle is from the rotation axis during one complete orbit.

The moment of charge of the beta particle, which is assumed to be solid, is

$$I_{bp} = \tfrac{2}{5} q_{bp} r_{bp}^2. \tag{17}$$

The average distance squared that the beta particle is from the moment axis during its orbit can be determined by assuming a circular orbit and realizing that the equation of the orbit if assumed in an x–y plane is

$$x^2 + y^2 = r_e^2, \tag{18}$$

or

$$x^2 = r_e^2 - y^2. \tag{19}$$

With this relationship, the average square distance is

$$d^2 = \frac{\int_0^{r_e} x^2 dy}{\int_0^{r_e} dy} = \frac{\int_0^{r_e} (r_e^2 - y^2) dy}{\int_0^{r_e} dy}. \tag{20}$$

Solving the two integrals gives

$$d^2 = \frac{r_e^3 - \frac{1}{3} r_e^3}{r_e}, \tag{21}$$

or

$$d^2 = \tfrac{2}{3} r_e^2. \tag{22}$$

This makes the product of the beta particle charge and its average distance squared from the moment axis during a complete orbit

$$q_{bp} d^2 = \tfrac{2}{3} q_{bp} r_e^2, \tag{23}$$

$$= \tfrac{2}{3} q_e r_e^2, \tag{24}$$

since $q_{bp} = q_e$. This is equal to the hollow-sphere electron charge moment. With this and the beta particle moment of charge, the actual moment of charge for the electron becomes

$$I_e = I_{bp} + q_{bp} d^2, \tag{25}$$

$$= \tfrac{2}{5} q_{bp} r_{bp}^2 + \tfrac{2}{3} q_e r_e^2, \tag{26}$$

$$= \left(\frac{2}{5} \frac{r_{bp}^2}{r_e^2} + \frac{2}{3} \right) q_e r_e^2. \tag{27}$$

I suspected that this expression corrected the hollow sphere approximation that the Bohr magneton makes for the magnetic moment. However, I did not have a value for r_{bp} to validate this.

4. Model Validation

To get some idea of what the radius of a beta particle was, I needed to know the radius of a proton. I speculated that the proton and the beta particle were both spheres made of the same material, so if I could determine the density of the proton, I could calculate the radius of the beta particle. However, there was no internet in 1979, and unless you had access to a good university library, finding specialized physics information such as the radius of a proton was nearly impossible. I had to devise a way of getting a good approximation of the proton's size. After kicking it around for a while, I decided to fall back on my nuclear engineering training to solve my problem.

In a light water reactor (LWR), the hydrogen nuclei (protons) in water molecules are used to slow down fast neutrons. When a fast neutron hits a proton, a large portion of its energy is transferred to the proton. Multiple collisions with protons effectively slow the neutron down. When a slow neutron collides with a proton, the neutron is said to be absorbed by the proton. What this really means is that the proton and neutron bind together to form a deuteron. In reality, given their similar sizes, it is hard to say whether the proton absorbs the neutron or vice versa. Anyway, the slow neutron absorption cross section of a proton, which can be thought of as the target area the proton presents to the neutron for interaction, is about 0.332 barns [6], which is 3.32 x 10^{-25} cm^2. Assuming that the neutron and proton have to touch for absorption to take place; the maximum distance the center of the neutron can pass from the center of the proton is one proton radius plus one neutron radius. If the neutron and the proton are essentially the same size, Fig. 2 shows that the proton's neutron absorption cross section is a circle that is approximately four proton radii in diameter. Since the cross section is 3.32 x 10^{-25} cm^2, if it is assumed to be a circle, its radius is 3.25 x 10^{-13} cm. This radius is twice the radius of the proton, making the proton's radius 1.625 x 10^{-13} cm, or 1.625 x 10^{-15} m.

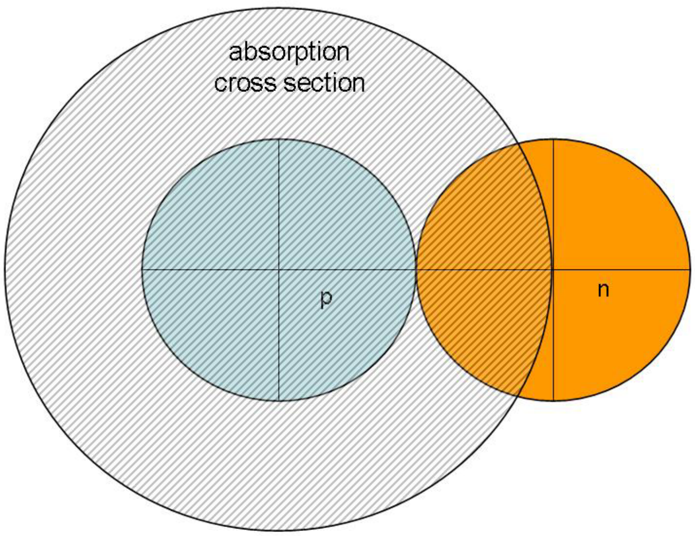

Fig. 2. Diagram illustrating how the proton's thermal neutron absorption cross section may manifest itself.

With a radius of 1.625 x 10^{-15} m, a spherical proton has a volume of 1.797 x 10^{-44} m^3. If the beta particles are spherical and have the same density as the proton (this was long before alpha-beta theory); then, since the proton is 1,836 times as massive as the electron, the beta particle volume should be 9.790 x 10^{-48} m^3. This would make its radius 1.327 x 10^{-16} m. Using the electron classical radius of 2.82 x 10^{-15} m [3] and substituting both values inside the parentheses in the adjusted electron moment of charge expression

$$I_e = \left(\frac{2}{5} \frac{r_{bp}^2}{r_e^2} + \frac{2}{3} \right) q_e r_e^2 \tag{28}$$

gives

$$I_e = 0.6675525\ q_e r_e^2\ . \tag{29}$$

Comparing this charge moment to the Bohr magneton charge moment of

$$I_B = \tfrac{2}{3} q_e r_e{}^2 \tag{30}$$

shows that

$$I_e = 1.00133 I_B. \tag{31}$$

This makes the adjusted electron magnetic moment 1.00133 times the Bohr magneton, which is very close to the actual multiplier of 1.00116. Because of this result, I concluded that the beta particle had to be the particle in orbit. That is why I modeled the electron the way I did in 1979, and why I carried the model over into the alpha-beta model when I developed it in 1993.

Using the alpha-beta model of the proton makes the result even better. If the proton is a sphere made of spheres packed together, then a packing factor [7] of about 0.74 must be applied to the proton volume to get the true volume of the beta particles. This means that the beta particles only take up about 1.330 x 10^{-44} m^3 of the proton's 1.797 x 10^{-44} m^3 volume. Also, this volume must be divided by 1,837 instead of 1,836, since there are 1,837 beta particles in a proton. These changes combine to make the beta particle volume 7.239 x 10^{-48} m^3 instead of 9.790 x 10^{-48} m^3, and its radius 1.200 x 10^{-16} m. This makes the electron moment of charge

$$I_e = 0.6673910\ q_e r_e^2\ , \tag{32}$$

and

$$I_e = 1.00109\ I_B\ , \tag{33}$$

so that

$$\mu_e = 1.00109\ \mu_B. \tag{34}$$

5. Conclusion

I realize that this could all be gibberish. Many proton radius measurements today report a radius about half the size I calculated in 1979, but the results do vary widely. Also, who knows what the real radius of an electron is? I used the classical radius, but it seems I've read somewhere that the actual radius may be smaller. Finally, the beta particle orbiting a neutrino is counter-intuitive. I would much rather it be the other way around, but that way does not support the magnetic moment calculation.

References

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