A Unit-Derivation for the Vacuum Field

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I derive a Lagrangian for all fields of force known, as well as all that could possibly be discovered in the future, and show that the sum of the fields of force in space equals the vacuum field of force and that this field can be measured in dimensions of kilograms per second. Using Gauge Theory and the Euler-Lagrange Method, I show that that which interacts in the vacuum field of force is velocity itself, that it is an interaction of motion and the foundation (or result) of all other fields of force. The consequences of this paper should allow one to better study the known fields of force, and because of the results of these mathematics, a more accurate Gauge Theory for the nature of fields of force may be understood.

1. Introduction

The author begins this mathematical proof with consensual definitions typically shared in the Field of physics, but often misunderstood by those lay to this study. The **Vacuum Field of Force** (the "Z Field") measurement derived in this proof is a measurement of the Field strength of the Field of Force associated with the *Vacuum Energy* (the "Z Energy"), that which is the sum of the energies of all Fields of Force in space (the Electric Fields, Magnetic Fields, **Gravitational Fields** ("Gravity"), other Gauge Fields, Fermionic Fields, Higgs Fields, etc.) [22]. *Z* Energy is distinct from a definition of **Zero-Point Energy** (the "*ZP* Energy" or "*ZPE*"), where the *ZP* Energy is the lowest energy a quantum mechanical **system** ("System") may have, the energy of its ground state. *ZP* Energy is well defined in the literature and is not the sum of all energies in space, as is the Vacuum Energy [21]. *ZP* Energy's associated **Zero-Point Field** (the "*ZP* Field") is the lowest energy state of a particular *field of force* ("Field"), also referred to as a "Virtual Field" or "Fundamental" or "Foundational Field", directly associated with the *ZP* Energy. Einstein and others had referred to the *ZP* Field as the "Z Field" [7], but this author differentiates the ground state with the '*P*' in "*ZP* Field" and "*ZP* Energy" from the vacuum "Z Field" and "Z Energy". While the Z Field may be directly associated with the *ZP* Field, the derivation of this paper considers the Z Field a non-conserved topological Field different in nature than that arising from quantum System processes, as opposed to that of the *ZP* Field, which is indeed by definition present in the makeup of all Systems.

The author derives a **Field lagrangian** (Lagrangian) for the *Z* Field's physical units that groups all the known and unknown Fields of space as a sum of compact **topological point-space elements** (the "Elements"), the whole of which comprises this Vacuum Field, an Element itself of the same **Field category** (the "Set") with the other Fields, only more fundamental. In other words, the sum of all Fields as Elements amounts to a single Element, this *Z* Field. Rearranging the topological derivation, the author produces the Lagrangian. The so-called Lagrangian (named after Joseph Louis Lagrange) of a dynamical System is described as a function that encapsulates the dynamics of the System [13]. For example, following the Principle of Least Action, credited to Pierre-Louis Moreau de Maupertuis, Leonhard Euler and Gottfried Leibniz who seemed to each have researched this principle independently [19, 8,12], classical mechanics defines the Lagrangian as kinetic **energy** ("Energy"), *T*, of the System minus its potential Energy $V : L = T - V$. Once the Lagrangian is identified, the equation of motion of the System can be attained with a replacement of the expression for the Lagrangian into the Euler-Lagrange Equation, which is satisfied by the following:

$$S(q)=\int_f^g L\left(t,q(t),q'(t)\right)\delta t \ , \tag{1}$$

where (1) satisfies the Euler-Lagrange Equation with function q,

$$q:\left[f,g\right] of\ group\, \mathbb{R}\to X \ ,$$
$$t\mapsto x=q(t) \ . \tag{2}$$

The manner characteristically entails calculating the Lagrangian (*L* equals *T* minus *V*) at several instants *t*, and drawing a graph of *L* against *t*. The area under the curve marks the **action** (the "Action") that interacts with the Field. For example, electric **charge** ("Charge") is the Action that the **electric Field** ("Electric Field") pushes or pulls on over some **distance** ("Distance"), resulting in electric **force** ("Force"). Euler's principle shows that nature always chooses the smallest Action when any different paths between point *A* and point *B* require a larger action than nature would ever select. This also implies that if a Lagrangian of all Fields could be derived to equal the *Z* Field, then the *Z* Field would always be associated with the least amount of Action for any given System it interacts with (even if it contains the greatest amount of Energy). This makes scientific sense according to Euler's Principle of Least Action, as it would be unlikely that anything could ever interact with an infinite magnitude Action (or possibly infinite, whatever is available in the universe) without completely destroying the other System upon interaction with it, especially for miniscule Systems. In other words, the magnitude of Energy in the *Z* Field should be extraordinarily large, but the Action should be extraordinarily small, smaller than any Action associated with any other Field.

A Lagrangian applicable to all the Fields and Forces in nature is a subject of research by a number of theoretical physicists [14], and an adequate Lagrangian to describe a single natural equation is the focal point of this proof. Perhaps some of the difficulties in describing such a Lagrangian for all Fields lies in the fact that scientists have yet to uncover all such possible Fields [10]. While there are typically considered four known fundamental Forces in nature, the unit measurement of any Force is always in **Newtons** (kilogram-meters per second squared or kg-m-s^{-2}), which is a measurement of how much Distance a given **mass** ("Mass") can transverse in a given amount of **time** ("Time") squared. While these Forces may drop off differently over Distance and their behaviors (and/or nature) may be distinct from one another, their units consistently remain the same, always in Newtons, which can make mathematic-originating discoveries in physics confusing and perhaps even dubious when only comparing Forces from different Fields and not the Fields themselves. For this reason, the author derives the Lagrangian for the Fields instead, as the units of any given Field are as unique as the Field's behavior and/or Action.

Other difficulties in extracting a Lagrangian for all the Fields perhaps lies too in the fact that while Fields may exist in mathematical Lie Groups (the electromagnetic Fields in the Abelian Circle Group U (1) for instance), others could theoretically exist in the form of Topological Groups, as the author will prove mathematically in this paper. Lie groups are relevant in Gauge Theory [11], as they can be studied using differential calculus by exchanging global objects with linear objects, as they are simply different forms of the other. This is only possible in terms of smooth manifolds, which Lie Groups are. Topological Groups cannot be approached in the same such ways. The Action of a geometric object interacting with a Lie Group (or a Lie Group acting upon a geometric object) produces a measure of inflexibility, producing mathematically definitive structure, which provide robust geometric constraints for precisely analyzing the manifold. When the manifold is a Field, the Action is the generator of symmetry for the System that interacts with the Field [1]. A Topological Group Σ on the other hand, is a Topological Space and Group such that the Group operations of product:

$$\Sigma\times\Sigma\rightarrow\Sigma:(x,y)\mapsto xy\ . \tag{3}$$

And taking inverses,

$$\Sigma\rightarrow\Sigma:x\mapsto x^{-1}\ . \tag{4}$$

Using product topology, Σ times Σ can be regarded as a **topological space** (the "Space"), but such a Sequence (a function over the domain of natural numbers: 0, 1, 2, 3, etc.) does not completely encode all the information about a function between Spaces [24]. A second limitation of natural number Sequences is that two conditions may not be equivalent in general for a map 'Θ' between Spaces 'X' and 'Y' [24]. In these cases, in order to generalize, a **topological net** ("Net") or Moore-Smith Sequence is employed. There are two co-implicating conditions for such a Net: 1) map 'Θ' is continuous and 2) given any point x in 'X', and any Sequence in 'X' converging to x, the **composition** ("Composition") of 'Θ' with this Sequence converges to '$\Theta(X)$'. Since the two conditions are implications of each other, only one or the other need ever be met to prove both. The primary obstacle in this study is that Spaces are almost never first countable [24].

In terms of Fields being applied in this proof, the Spaces described are understood in the Field's units (r, r^2, r^3, r^4, etc., where r is Distance), a continuous Sequence of raised natural numbers that the author will present. Thus, in order to apply a Net to express all Fields in order to meet condition one above (an infinite number of possible Fields), some amount of assumption is required to construct such a Lagrangian. However, as condition two is met, because condition two implies one, the assumption of an infinite number of Fields that can be described topologically (as well as an infinite number of Lie Fields) is eliminated and replaced with mathematical theorem.

Conjecture 1

An infinite number of unitary groups ("Unitary Groups") $U(\infty)$ for a unitary matrix exists over the degree of n to infinity for all integers n in U(n).

The Circle Group (U of one) of 1×1 unitary matrices involving electrodynamic Systems of electricity and magnetism, having one boson (the photon) is unique, but not mutually exclusive to SU of two, its **subgroup** ("Subgroup") of degree two (involving the **electroweak Field,** "W Field"), having three bosons, or SU of three, a Subgroup of degree three (involving the **nuclear strong Field,** "S Field"), involving even more bosons or any other Unitary Group. In other words, there are an infinite number of Unitary Groups and an infinite number of Fields. This also goes for Groups of n equal to zero and n to minus infinity.

Conjecture 2

The infinite number of Unitary Groups are each modular to three Fields for each n in U of n.

The Circle Group U of one can only describe the **pseudovector magnetic Field** (the "B Field"), the **vector magnetic Field** (the "A Field") and the Electric Field (the "E Field"). In the same, SU of two can only describe three Fields as well (the **electroweak Field** (the "W Field") being one of its three) as can SU of three only describe three (or any other)—the number of Fields for each Unitary Group is modular. This also goes for groups of n equal to zero and n to minus infinity, except that it could be said that there is a more infinite number of Fields than Groups, as there are three Fields to each Group to infinity.

Conjecture 3

An infinite number of Unitary Groups and Fields would be isomorphic to a group of numbers on a separate number plane, increasing in complexity as $n\rightarrow\infty$.

The number plane on which each Field is described increases in complexity as n increases in value. In other words, as is typically known, U of one is isomorphic to the group of complex numbers, thus diffeomorphic to the two-dimensional sphere, SU of two is isomorphic to the group of quaternions of norm one, thus diffeomorphic to the three-dimensional sphere—but that this planar complexity increases infinitely as n tends toward infinity. This also is true for groups of n equal to zero and n to minus infinity.

Conjecture 4

The modularity of the Field variables of each Unitary Group to infinity is mutually complimentary to the variables of each Field space to infinity.

If each Unitary Group is limited to three Field variable congruence relations a, b and c (where Fields $B\equiv a$, $A\equiv b$ and $E\equiv c$

in *U* of one), then so too are the number of possible unit variables raised to some power in each Field limited to three, which is how there could mathematically amount a more infinite number of Fields than Groups. For example, the *B* Field is in kg-C^{-1}s^{-1}, the *A* Field is in kg-m-C^{-1}s^{-1} and the *E* Field is in kg-m-C^{-1}s^{-2}; since Distance, Charge and Time are continuous variables to some power of *a*, *b* and *c*, a Field in these Groups cannot be described as Mass to some power other than one. While *a*, *b* and *c* are continuous, *a* corresponds to one Field unit (Time for instance) and one Field in a Unitary Group (the *B* Field for instance), *b* corresponds to another Field unit (Charge for instance) and one Field in a Unitary Group (the *A* Field for instance) and *c* corresponds to another Field unit (Distance for instance) and another Field in the Unitary Group (the *E* Field for instance).

This requires that Mass exists in all Field units, being able to reduce the Fields to individual points from Spaces, but only ever raised to the power of one, a **constant function** ("Constant Function", in that its values remain the same for any argument in its domain) if it is in any of them (which it is), as it is analogous to the Unitary Group of the measurement, the Group of which it comprises. Thus, all Fields in a function governing all of them must be **homotopic** ("Homotopic"), meaning that the Field can be stretched, deformed or scaled to take on different shapes, excluding punctured holes, tears or the like) to the form of Mass in order to be **contractible** ("Contractible"). Specifically in terms of this paper, Contractible means that even as a Space, a Field can be mathematically scaled down to a single point in order to comprise yet another Space, and it will be proven in this paper that all the Fields to infinity can be morphed in this way. Also, Mass needs to be only ever raised to the power of one in such a function for the function to be considered **null-homotopic** ("Null-Homotopic"), always a Homotopic equivalence [3], as it also will be proven to be; and it follows from the definitions governing morphisms that a space *X* is contractible if and only if the identity map from *X* to itself is null-homotopic [3], as well as when this author approaches the Unitary Groups in terms of topological products ("Products") later in this paper, from Tychonoff's theorem any Product of compact spaces is also compact [24]. Many of these results ride on the fact that Mass can only be raised to the power of one in terms of the functions to follow.

Lastly, all Fields must involve in its dimensional units Distance, Charge and Time raised to some integer (in terms of the *B* Field, *kg* can be considered raised to one, *m* raised to zero, *C* raised to minus one and *s* raised to minus one, as all are integers the same as the groups' degrees), such that the product of powers for each Field are Homotopic to the power Mass is raised to (the value of one), else a congruence relation between the Unitary Groups and the powers of the Fields could not be made. As a result, the complexity of the Fields increases or decreases modularly in relation with the groups, and the groups increase or decrease in terms of the number planes (real numbers $\mathbb{R}$, complex numbers $\mathbb{C}$, quaternion numbers $\mathbb{H}$, etc.), as do their isomorphic or diffeomorphic relationships. Thus, *a*, *b* and *c* increase as powers in direct relation with the Unitary Group degree, but only over three variables: seconds to the plus or minus *a* (Time to the power of plus or minus *a*), Coulombs to the plus or minus *b* (Charge to the power of plus or minus *b*) and meters to the plus or minus *c* (Distance to the power of plus or minus *c*).

2. Application of a Dimensional Sequence to the Unitary Groups

In order to solve for the units of the *Z* Field, the author uses three known conserved parameters: 1) the **electron rest mass** ("Electron Mass"), 2) the **classic electron radius** ("Electron Radius") and 3) the Electron Charge. By considering the Electron Radius to be negative, being that it is in a region closed (closed only in terms of space, not force or other measurements) to all the external possible Spaces (the other Fields of space), one can calculate a value of Time to be applied to the above Field Sequences. In order to calculate an adequate Time value to be applied to the above Sequence, such that all the Energies of all the external Fields are equal (this calculation only intends to solve for the units of the *Z* Field and not its true to nature numerical value if indeed it could have one), the author considers a measurement of some arbitrary wave propagating from the electron's center radius to its surface. With this arbitrary wave propagation Velocity, one can extract a measurement of Time. The simplest value for this Velocity is $v = -1$ m-s^{-1}, negative as the wave given to propagate exists within the Electron Radius, involving a negative Distance in the same way the negative Electron Radius is used. This also permits the Specific Energy of the external Fields to be $v^2 = 1$ m^2-s^{-2}, positive, thereby allowing the electron to interact with any of these Fields (all possible conceivable Fields toward infinity) being that all Frequencies of all energies could resonate with the electron, as the Charges, Energies, Distances, Masses and Frequencies would all be equal, just not raised to some varying power.

By raising the values of the units of any Field to the power of a sequence that is not first countable:

$$p(a,b,c) = a+1, b+1, c+1, a+1, b+1, c+1, \dots, \tag{5}$$

where *a*, *b* and *c* are natural numbers.

Such **set** ("Set") *X* of Spaces, of which the Sequence in it should converge to some Element *x* and the Composition of 'Θ' with this Sequence converges to '$\Theta(X)$', as describer earlier in order to meet the conditions of the Net, the Sequence *p* of *a*, of *b*, of *c* can be integrated as a power Sequence in the following topological Set *X* comprised of Elements

$$x = \mathrm{M}\mathrm{t}^{-a}\mathrm{e}^{-b}\mathrm{r}^{c} \quad : \quad \begin{matrix} a \geq b > a-2 \\ b \geq c > b-2 \end{matrix} \tag{6}$$

where M is Mass, t is Time, e is Electric Charge and r is Distance with respect to Sequence *p* of *a*, of *b*, of *c*, though expressed such that the natural number arguments *a*, *b* and *c* are conditional powers of given known conserved values (Electron Mass, Electron Charge, etc.), efficiently eliminating Sequence *p* from the rest of this paper—and any x is a Field (in $kg\ s^{-a}\ C^{-b}\ m^{c}$) in the *X* Set of Fields, the Lie Fields, of which will be treated as Spaces. Essentially, the raised *a*, *b* and *c* in (6), following the conventions to the right of the colon, increase or decrease in value the same as (5) if taken in regular intervals. But in (6), one is able to expand on the Sequence or work outside of the Sequence altogether.

While it may be difficult for some readers to understand how a Field could be described as a Space, having the dimensional units of Mass, Time and Charge (Distance may seem more straightforward) in its dimensions, some clarification could be required. While a Field is indeed a Topological Space [4], its units are only measurements of that Space. Thus, the definition of a Field's unit measurements in this context is given below, which already describes all known Fields (B Field, A Field, E Field, etc.).

Field of Force Measurement: The measurement of the amount of *Space* (r to some power) a given *Mass* (M), having a certain amount of *Electric Charge* (e to some power) transverses from a *Field of Force* (F_N) in a certain amount of *Time* (t to some power).

In this sense, for the remainder of this paper, all Spaces will refer to Fields, and vice versa following the above definition. Additionally, Elements of x can be added together to make larger Spaces, the Unitary Groups, which can then be multiplied and applied to (3) and (4) and more, as will be shown.

It will also be described how any Space x from (6) can be mapped to any Space y:

$$y = \mathrm{Mr}^{-a}\mathrm{e}^{b}\mathrm{t}^{c} \quad : \quad \begin{matrix} a \geq b > a-2 \\ b \geq c > b-2 \end{matrix}\,, \tag{7}$$

where y is any Field Element not in the Set of Lie Groups X; rather, Y, the Set of Topological Groups.

As some readers may recognize, if integers were to be applied as arguments to (7), then the two equations describing either type of Field would amount to the same values, just that all the Fields in Set Y would require a different Sequence from (5), a Sequence that subtracts one from each variable instead of adding one. But this is not a function over the domain of Integers; rather a topological Sequence over the domain of the natural numbers. As hinted at earlier in terms of the Fields being Homotopic to Mass, one of the primary results of this paper is that it will describe how all the Elements x are Homotopic to Elements y, meaning that any Field from the pseudovector magnetic B Field to the nuclear strong S Field and everywhere in between (gravitic, electric, etc.) are Homotopic to one another; that is, while they may not share all the information between each other, there exists a map Θ that links their topological properties (topological invariants) enough so that one can be morphed into another using some mathematical or physical means (in terms of physics the means might be high energies, resonance, or something to that effect). And this is admittedly a strong statement, but it should be easy enough to prove the opposite be it so, as to prove that two or more topological spaces are not homeomorphic, mathematicians find it adequate to locate but a single **Property** not mutually shared between the Spaces [24]. This author seeks to demonstrate as a consequence to the proof that all such Properties are shared between the various Fields to infinity, and that such a deviating Property does not exist in terms of the mathematics presented.

To think of any given Field x or y as a point in Space X or Y, even though a Field (even a non-compact Field) takes up space, it becomes manageable to visualize (at least mathematically) the entire universe as a single Space that is made whole by the existence of the sum of all these individual points, these individual Fields, and that this entire space of the universe can be differentiated by two different types of Spaces: 1) smooth, symmetric manifold space X and 2) asymmetric space Y. And of course while some (if not most) Fields extend infinitely (Electric Field, Gravity, etc.), some do not (Nuclear Strong Field); thus, because all the Fields will be shown to be Homotopic to one another, the infinite extensions of the Field lines can be restricted in order to prove the mathematics. In other words, all the Fields can be confined and isolated mathematically (and many experimentally) in order to compare their topological Properties, to see how they all contain the same Sequence, the same Properties. Again, while the Sequence above does not contain all the information between X and Y, the Net will because collections of open Sets in Spaces are much like directed Sets in behavior [17].

Again, the author should restate the meaning behind what will follow: the sum of $X + Y$ will equal the Space of the universe (the entire vacuum), which will have the consequence of being both infinite in some respects, but compact in general, and the sum of all Elements x (the Lie Fields) fill Space X and all Elements y (the Topological Fields) fill space Y—and the two spaces take up all the Space in the universe. The proof will show that the Space of the entire universe is the Vacuum Field Z (as is defined above) and that its SI (Standard International) units can be measured in $\mathrm{kg\text{-}s^{-1}}$.

In order to meet the required conditions of a Net, as well as the understood physical definition of the Z Field Energy being equal to the sum of the Energies of all Fields in in the universe, one can first begin by summing all the Lie Fields to Set X,

$$\begin{aligned} X = {} & \mathrm{Mt^{-1}e^{-1}r^{0}} + \mathrm{Mt^{-1}e^{-1}r^{1}} + \mathrm{Mt^{-2}e^{-1}r^{1}} \\ & + \mathrm{Mt^{-2}e^{-2}r^{1}} + \mathrm{Mt^{-2}e^{-2}r^{2}} + \mathrm{Mt^{-3}e^{-2}r^{2}} + \ldots, \end{aligned} \tag{8}$$

which does leave one natural argument out (zero, see below), and can be expressed as follows:

$$X_w = \sum_{w=1}^{\infty} \mathrm{Mt}^{-a}\mathrm{e}^{-b}\mathrm{r}^{c} \quad : \quad \begin{matrix} a+b+c = w+1 \\ a \geq b > a-2 \\ b \geq c > b-2 \end{matrix}\,, \tag{9}$$

where a, b and c become variables dependent upon independent w, taking the sum of all natural arguments for (6) in order to infinity with the single exception of w = 0 (it will be discussed in the next pages why natural zero is not an argument of this function).

And one can do the same for all the Topological Fields in Y.

$$\begin{aligned} Y_w = {} & \mathrm{Mr^{0}e^{0}t^{0}} + \mathrm{Mr^{-1}e^{0}t^{0}} + \mathrm{Mr^{-1}e^{1}t^{0}} + \mathrm{Mr^{-1}e^{1}t^{1}} \\ & + \mathrm{Mr^{-2}e^{1}t^{1}} + \mathrm{Mr^{-2}e^{2}t^{1}} + \mathrm{Mr^{-2}e^{2}t^{2}} + \mathrm{Mr^{-3}e^{2}t^{2}} + \ldots \end{aligned} \tag{10}$$

which equally can be expressed as,

$$Y_w = \sum_{w=1}^{\infty} \mathrm{Mr}^{-a}\mathrm{e}^{b}\mathrm{t}^{c} \quad : \quad \begin{matrix} a+b+c = w-1 \\ a \geq b > a-2 \\ b \geq c > b-2 \end{matrix}\,. \tag{11}$$

In this way, all possible Fields x and y (except for one) to infinity are mapped into two separate Sets X and Y, $x \to X, y \to Y$ though not yet mapping X to Y, where the single eliminated Field exists at argument w = 0 for either (9) or (11), as considering zero to be in the domain of natural numbers (zero being a natural number); these functions apply to Spaces. However, as an important note, if Elements x_0, y_0 were to be applied to either sum at w = 0, then their arguments to either function would be equal ($x_0 = y_0$) for any conserved Mass, Distance, Charge or Time; however, in doing so, the values of the sums of both *X* and *Y* would amount to something different. While that certainly could be done, these functions are leading toward the Lagrangian, and thus $w = 0$ is not considered at this time for reasons to be described later.

In short, the above functions in order of the natural numbers not only solve for all the known Fields (the *B* Field arises at argument $w = 1$ in X_w, the *A* Field at $w = 2$, and the *E* Field at w = 3), but all the possible Fields, whether or not the Field is a smooth manifold, studied with differential calculus or not, which begins to tend toward the result of the final proof to be presented: the dimensions of the *Z* Field are in SI Units kg-s^{-1}, as the dimensions at $w = 0$ (if it were applied) would amount to measurements in $\text{kg-m}^0\text{-C}^0\text{-s}^{-1}$ which equals kg-s^{-1}.

Next, given the function

$$f_x(w) = 5 \times 10^{3+75w/14} , \tag{12}$$

which is over the domain of natural numbers, and some constant *k*, where *k* = 6.271921…, one possesses all the mathematic tools to describe and prove the **Z Field Θ Theorem**.

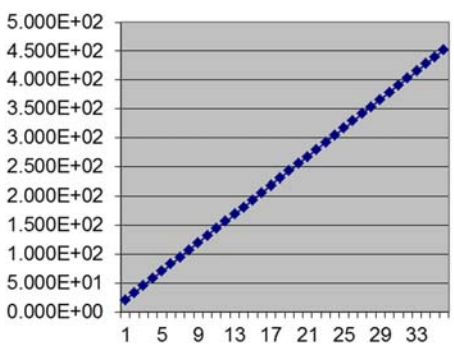

Fig. 1. Graph of $\ln f_x(w)$, showing its linearity

3. Z Field Θ Theorem

$$\Theta = |Y_w| + |X_w| - \left| f_x(w) \exp\left(k + \ln\left(|X_{w-3}| \div f_x(w-3)\right)\right)\right| , \tag{13}$$

where ln is the natural logarithm and the w minus three requires the value to be taken from three arguments back of functions $f_x(w)$ and X_w, which suggests that the second term in the second absolute value in (13) is the part of Net Θ that allows mapping from *X* to *Y*, and that the Field function X_w is built modularly from Fields in the previous ("previous" in Sequential definition) Unitary Group (three places backward in the Sequence), as will be described in depth following the proof.

It is for this reason that the value of the functions both at X_{w-3} and $f_x(w-3)$ are required in order to solve for any next value of X_w. While derivations could hypothetically be made to provide a standalone function for each without references three arguments back, the author leaves (13) as it is in order to show the modular-three nature of the Field progressions to follow. Also, because the dimensions of the *B* Field, the *A* Field and the *E* Field are already well known in physics [9], the fourth Field of X_w in *SU* of two can be solved from the value of the *B* Field, as well as any that follow (this two will be explained in depth during and following the proof).

The meaning of (13), coupled with the incorporated results of this paper, is that the *Z* Field is a **zero object** (or null object, referred to in this paper as a "Null Element", precisely a null point space Element), meaning that it is both the **initial object** ("Initial Element", a point space Element) and **terminal object** ("Terminal Element", also a point space Element) of *X* and *Y*. Said Initial Element of a category *X*, in this paper a Space, is Element *I* in *X* such that for every Element *x* in *X*, there exists a single morphism $I \to x$, and *Z* is terminal if for every Element *x* in *X* there exists a single morphism $x \to Z$ [2]. The result of (13) states that the Initial Element equals the Terminal Element (thus a Null Element), as this proof will demonstrate, and that the Null Element is the *Z* Field.

For a more complete meaning of the significance of (13) before the derivation begins, the result also suggests that $\Theta : X \to Y$ is a **Forgetful** (and **Faithful**) **Functor**, thus an Injective Function, preserving distinctness, never mapping distinct Elements of *X* to the same Element of *Y* [2]. In other words, every Element of the function's codomain is mapped to at most one Element of its domain, which are at **Basepoints** x_0 and y_o.

Electron Mass (kg)	Electron Charge (C)	Electron Radius (m)
9.10938… E-31	-1.60217… E-19	-2.18794… E-15

Table 1. Values of Electron Parameters

Using the generally accepted SI Units values of the electron from Table 1, one can begin to solve for an arbitrary Time value *t* to apply to (13). Using the example earlier of the propagating wave from the electron's center to its surface (but not beyond) traveling at minus one meter per second (a negative Velocity by convention, as the wave is traveling from the inside of the radius to the surface), whose negative value becomes equal to our negative electron radius (though equal only in value, not dimensionally). This gives

$$\text{Wave Time } t = \text{Velocity } v \times \text{ Electron Radius } r = -1 \times r , \tag{14}$$

which provides us four values to apply to (13), referencing three actual measurements and an arbitrary Time value, a directed value that allows all Field frequencies and energies in (13) to be equal, allowing the electron to hypothetically interact with any infinite number of Fields while in resonance with them—for the purposes of this mathematical proof.

M	e	r	t
9.109… E-31	-1.602… E-19	-2.187… E-15	2.187… E-15

Table 2. Values of Electron Parameters and Time

4. Proof of the *Z* Field Θ Theorem

Solve for the first value of X_w using the given conserved values from Table 2 and equation (9), which provides the dimensions of the pseudovector magnetic *B* Field. One gets

$$X_1 = \left(9.109\ldots\times10^{-31}\,\text{kg}\right)\left(-2.187\ldots\times10^{-15}\,\text{m}\right)^0 \cdot\left(-1.602\ldots10^{-19}\,\text{C}\right)^{-1}\left(2.187\ldots\times10^{-15}\,\text{s}\right)^{-1} = 2.598\ldots\times10^{3}\ \text{kg/C-s} \tag{15}$$

Then solve for the next value of X_w, which has the dimensions of the vector magnetic *A* Field, and add it to the first value. This gives

$$X_2 = X_1\ \text{kg-m/C-s} + \left(9.109\ldots\times10^{-31}\,\text{kg}\right)\left(-2.187\ldots\times10^{-15}\,\text{m}\right) \cdot\left(-1.602\ldots10^{-19}\,\text{C}\right)^{-1}\left(2.187\ldots\times10^{-15}\,\text{s}\right)^{-1} = 2.598\ldots\times10^{3}\ \text{kg-m/C-s} \tag{16}$$

Then solve for the next value of X_w, which has the dimensions of the electric *E* Field, and add it to the first and second values. This gives

$$X_3 = X_1 + X_2\ \text{kg-m/C-s}^2 + \left(9.109\ldots\times10^{-31}\,\text{kg}\right)\left(-2.187\ldots\times10^{-15}\,\text{m}\right) \cdot\left(-1.602\ldots10^{-19}\,\text{C}\right)^{-1}\left(2.187\ldots\times10^{-15}\,\text{s}\right)^{-2} = 5.991\ldots\times10^{-12}\ \text{kg-m/C-s}^2 \tag{17}$$

Continuing all the way to infinity,

$$X_\infty = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + \ldots = \left[\text{Non-Generalized}\right]\text{kg-m}^{\infty}\text{C}^{-\infty}\text{s}^{-\infty} \tag{18}$$

where the function of this Sequence of Spaces does not generalize on its own without Net Θ, but all the Lie Fields are indeed being summed.

One gets the first few values as shown in Table 3.

w	X_w
1	-2.598… E+03
2	-2.598… E+03
3	5.991… E-12
4	-1.621… E+22
5	-1.621… E+22
6	3.355… E+07
7	-1.012… E+41
8	-1.012… E+41
9	2.127… E+26
10	-6.318… E+59

Table 3. Values of X_w for the Electron Example

Next, solve for the first value of Y_w. For Y_1, we get

$$Y_1 = \left(9.109\ldots\times10^{-31}\,\text{kg}\right)\left(-2.187\ldots\times10^{-15}\,\text{m}\right)^0 \cdot\left(-1.602\ldots10^{-19}\,\text{C}\right)^{0}\left(2.187\ldots\times10^{-15}\,\text{s}\right)^{0} = 9.109\ldots\times10^{-31}\,\text{kg} \tag{19}$$

where the dimensions amount to kilograms, directly stating that Mass is a Topological Space—a Field in and of its self. (19) is the first strong result that begins to show how all Elements in the Groups of Spaces (the Fields), are Homotopic to Mass to the power of one, a constant function they map to, as Mass has resulted as the first value of Y_w, thus already allowing any Elements of Y_w that follow to be Contractible, as they all will be Homotopic equivalents.

Next solve for Y_2 and add it to the first. This gives

$$Y_2 = Y_1\ \text{kg} + \left(9.109\ldots\times10^{-31}\,\text{kg}\right)\left(-2.187\ldots\times10^{-15}\,\text{m}\right)^{-1} \cdot\left(-1.602\ldots10^{-19}\,\text{C}\right)^{0}\left(2.187\ldots\times10^{-15}\,\text{s}\right)^{0} = -4.163\ldots\times10^{3}\,\text{kg/m} \tag{20}$$

where the dimensions result to **Density** (*kg m*-1), also an Element of *Y*—a Field in and of itself.

Continue in the same way to infinity. One gets

$$Y_\infty = Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + \ldots = -4.163\ldots\times10^{-16}\ \text{kg-C}^{\infty}\text{s}^{\infty}\text{m}^{-\infty} \tag{21}$$

where this function does converge almost instantly, evident at even the second value of the function.

At this time the author reconsiders x_0, y_0. As mentioned earlier there remains one more combination of natural numbers *a*, *b* and *c* that would meet the condition of both (6) and (7). For (6), the following for $x \to X$ is true but not yet considered in the summation of (9):

$$\begin{array}{l} a=1 \\ b=0 \\ c=0 \end{array} : \begin{array}{l} a \ge b > a-2 \\ b \ge c > b-2 \end{array} \tag{22}$$

Thus,

$$x_0 = \left(9.109\ldots\times10^{-31}\,\text{kg}\right)\left(-2.187\ldots\times10^{-15}\,\text{m}\right)^0 \cdot\left(-1.602\ldots10^{-19}\,\text{C}\right)^{0}\left(2.187\ldots\times10^{-15}\,\text{s}\right)^{-1} = 4.163\ldots\times10^{-16}\,\text{kg/s} \tag{23}$$

where the value is the positive counterpart of (21), but in kilograms per second.

But in terms of (7) and (11), in order for $y \to Y$, one runs into evidence that these results describes Spaces locally homeomorphic to Euclidean space with the first condition of Y_w: $a+b+c = w-1$, but not necessarily Hausdorff [24]. When w = 0, the value of natural numbers $a+b+c=-1$, which is not possible as written, as all natural numbers are positive and a sum of positive numbers cannot equal a negative number arithmetically. One instead gets what is referred to as a "signed zero" [16], which can be seen by multiplying both sides of the condition by minus one, which gives $-(a+b+c) = -w+1$. When w equals zero in this case, one is tempted to reduce a minus zero to just zero. In mathematics, it is a usual axiom of a manifold to be a Hausdorff space, and this is assumed throughout geometry and topology [24]: "manifold" *means* "(second countable) Hausdorff manifold". However, in general topology, this axiom is relaxed, and one studies *non-Hausdorff manifolds.* In arithmetic

$-0=+0=0$, but in general topology this amounts to Space y_o being a **quotient Space** ("Quotient Space"), an identifying Space [24]. It can be understood in terms of a "gluing together" of certain points of a given space. The two points are henceforth understood as a single point, or wedge point, and this is exactly what this author is organizing, gluing X_w to Y_w to a single point Space.

What the first condition of Y_w presents (whether or not $w=0$) is that the sum of natural numbers *a*, *b* and *c* equals a minus *w* (an integer) plus one (another natural number). To treat *w* as a natural number (simply a zero, rather than a minus zero) would result in lost information about the topological Space when *w* = 0. This lack of information if followed through would amount to a second Field of density *D* in *Y*, which is the Space that already results at *w* = 2. If allowable, this would be inaccurate at worst, redundant at best. The only possible mathematical approach is to treat the condition as $-(a+b+c)=-w+1$ instead of $a+b+c=w-1$ and then add *w* to both sides: $-(a+b+c)+w=1$, which carries over the equivalence and does not cause a loss of information. In the end, it results in more information with the integer number minus one that allows the equation to be calculated instead of merely eliminating y_0 as an Element of *Y*, where it does rightfully belong. Thus, when *a*, *b* or *c* is a natural number, it works fine to meet the first condition accurately, so long as one remembers to bring the integer minus one into the Y_w later in the same manner as that which follows below:

$$\begin{array}{lcl} -1\times a=0 \Longleftrightarrow \mathbb{N}\{a\}=0 & & -(a+b+c)+w=1 \\ -1\times b=0 \Longleftrightarrow \mathbb{N}\{b\}=0 & : & \mathbb{Z}\{a\}\geq\mathbb{Z}\{b\}>Z\{a\}-2 \\ -1\times c=-1 \Longleftrightarrow \mathbb{N}\{c\}=1 & & \mathbb{Z}\{b\}\geq\mathbb{Z}\{c\}>Z\{b\}-2 \end{array} , \qquad (24)$$

where the second and third condition of (24) now includes integer variables, as *a*, *b* and *c* are dependent variables to *w* in Y_w. Thus, multiplying a minus one by *w* in the first condition results in real conditions for any dependent upon *w*.

Raising Distance *r*, Charge *e* and Time *t* in (7) to the product of a real minus one times the values of naturals $a=0$, $b=0$ and $c=1$ in (24) simply reduces the power of Charge and Distance to zero and inverts Time in *y*. This gives

$$\begin{aligned} y_0 &= \left(9.109\ldots\times10^{-31}\,\text{kg}\right)\left(-2.187\ldots\times10^{-15}\,\text{m}\right)^0 \\ &\cdot\left(-1.602\ldots10^{-19}\,\text{C}\right)^0\left(2.187\ldots\times10^{-15}\,\text{s}\right)^{-1} \\ &= 4.163\ldots\times10^{-16}\,\text{kg/s}. \end{aligned} \qquad (25)$$

Then subtracting (25) from (23), one gets

$$x_0 - y_0 = 0 \ . \qquad (26)$$

They are equivalent, thus

$$x_0 = y_0 : x \mapsto y \ .$$

But the author only still considers *x* and *y* as individual Spaces in *X* and *Y*, not Elements of the summations of X_w or Y_w. This is only permitted when there exists a third summation Z_w with a single Element [24]: the value of the Quotient Space x_0, y_0. This Space *Z* becomes a **null object** ("Null Object") for the entire category of Field Elements, but a Space in a Topological Group (as opposed to the Lie Groups, which will be described). In other words, all the Lie Fields become Elements of the Lie Groups (including the Unitary Groups commonly known), and all the Topological Fields are Elements of the Topological Groups. And according to the conjectures earlier, there should be just three Elements to any one group. This allows there to be three Sets of Elements, but only ever two Sets of Groups, which is not readily obvious until considering the following product of all Unitary Groups to infinity, but the author can state ahead of time that the single Element in Space *Z* must belong to a Topological Group and not a Lie Group, as will become considerably obvious once the author includes below a visual figure and presents two product equations.

Referring back to (3) and (4) and the discussion immediately following (mentioning how Σ is both a Group and a Space), the author will now discuss and denote any such Groups as *U* instead of Σ, as the paper refers to the Unitary Groups, which are typically denoted *U* or *SU* for subgroups (the author will no longer use the denotation *SU* either). These Groups according to the conjectures earlier are considered to be infinite in number $U(n)$, and the Space comprising each of these groups are limited to three unique **subspaces** ("Subspaces") each, which are the unique Fields to infinity *x* and *y* (*D*, *M*, *Z*, *B*, *A*, *E*, etc.). Thus, since the unique Fields (the smaller Spaces) are added together to make a larger space, a product function is easy to construct following that which has already been provided in this paper.

Begin by adding the first three Lie Fields together. The values of (6) are to be used, but the order can be mapped using the *w* from (9). One gets a zero from their sum,

$$x_1 + x_2 + x_3 = B + A + E = 0 \, , \qquad (27)$$

where the sum of the *B* Field plus the *A* Field plus the *E* Field in the Circle Group U of one has no value, still referring to our conserved electron values.

Continuing for *U* of two (the Unitary Group containing the Electroweak Field *W*), one gets the same.

$$x_4 + x_5 + x_6 = N + P + W = 0 \, , \qquad (28)$$

where the *N* (not to be confused with the symbol *N* used later in this paper for Force; *N* for this Field will be denoted with a different symbol in later publications in order to better avoid confusion) and *P* are Fields not discussed any more in depth in this paper. *W* is the Electroweak Field.

Obviously multiplying (27) by (28) would also equal zero. Next taking all the products to infinity, one gets

$$U_x(n) = (x_1 + x_2 + x_3)(x_4 + x_5 + x_6)(x_7 + x_8 + x_9)(\ldots) = 0 \ , \qquad (29)$$

which can equally be expressed as

$$U_x(n) = \prod_{n=1}^{\infty}\left(x_{3n-2} + x_{3n-1} + x_{3n}\right) = 0 \ , \qquad (30)$$

where the product of all Lie Groups equals zero.

The same can be done for the Topological Groups, first beginning with x_0, y_o (which will be considered a Field in its own right at the end of this proof, denoted *Z* for now), then Mass and then Density, which are the first three fields in this.

$$y_0 + y_1 + y_2 = Z + M + D = 8.875\ldots\times 10^{-31} > 0 \tag{31}$$

Not a constant function, but the infinite product does converge.

$$U_y(n) = (y_0 + y_1 + y_2)(y_3 + y_4 + y_5)(y_6 + y_7 + y_8)(\ldots) = 0 \ , \tag{32}$$

which can equally be expressed as

$$U_y(n) = \prod_{n=1}^{\infty}(y_{3n-3} + y_{3n-2} + y_{3n-1}) = 0 \ , \tag{33}$$

where the product of all Topological Groups equal zero.

$$U_x(n) = U_y(n) \ , \quad x_0 = y_o \ ,$$

$$U_x(n) \times U_y(n) \to U_x(n),\, U_y(n) : (x,y) \mapsto xy \ . \tag{34}$$

Thus,

$$U_x(n), U_y(n) : y \to X \ . \tag{35}$$

While the *Z* Field Theorem has not yet been proven, the four earlier conjectures indeed are, as everything from their infiniteness to their modular connection with the congruencies between the Groups and the Fields comprising them are satisfied in (34) and (35). Additionally, the results of the conjectures being proven will become evident and useful upon moving forward. While it yet may be premature until the remainder of the proofs are presented, Fig. 2 provides a diagram of how the entire Fields Category *Grp* is beginning to reveal its structure to infinity, as the author understands it based on the math thus far.

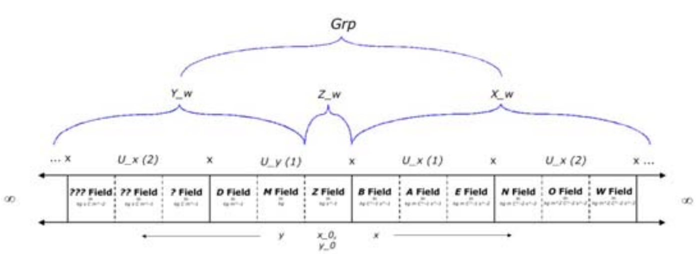

Fig. 2. Diagram of the Field Grp

The diagram is centered on the *Z* Field, which is the only thus far unproven connection. While there indeed exists a Field in that location, gluing the two Sets (the large Spaces comprising all the individual Spaces) using x_0, y_0, and that it is the Zero Object of *Grp*, it could not be entirely certain to any reader which field that is without the proof of the *Z* Field Θ Theorem; in other words, it cannot be certain at this point that this Field is the true Vacuum Field *Z* without that which follows.

In order to move forward toward proof of the *Z* Field Theorem, one can next apply the values from (19), (20) and (23) or (25) to second term of the second absolute value of (13), as all elements of $y \to X$. Apply the values *k*, $f_x(w-3)$ and (20) to solve for a dimensionless value equal to the physical *B* Field for *w* equal to one for X_w and two for Y_w. Because neither is first countable, either can be solved using a derivation of the other and have all points shared somewhere in both functions. Having the same values, three places backward in the continuous Sequence, a new function can be utilized to solve for the first couple values of X_w. One can use the sum of Y_w up to Field Density *D* in (20) to solve for (36). This gives

$$\begin{aligned}\pi_1 &= f_x(1)\exp\left(k + \ln\left(\left|Y_2\right| \div f_x(-2)\right)\right)\\ &= f_x(1)\exp\left(6.271\ldots + \ln\left(\left|-4.163\ldots\times 10^{-16}\right| \div 9.653\ldots\times 10^{-8}\right)\right)\\ &= 2.598\ldots\times 10^{3}\ \text{dimensionless}\ ,\end{aligned} \tag{36}$$

where the absolute value of $X_{w-3} = Y_2$ which is the sum of Mass plus Density in Y_w, and so its value is used to solve for the value π_B which equals that of the *B* Field, equal in value, not dimensions. The method above (and how it reduces to a dimensionless value follows only from the fact that (36) is an element of infinite functions (30) and (33), both amounting to zero. The Buckingham π Theorem [5] states that different systems that share the same description in terms of dimensionless numbers are equivalent, if the following is true:

$$\phi(X_1, X_2, \ldots, X_\nu) = 0 \ , \tag{37}$$

where the X_w are the ν (Greek symbol *nu*, not to be confused with a lowercase *v*, which the author retains for Velocity) physical variables (infinite in this case), and they are expressed in terms of κ independent physical units (also infinite in number), then the above equation can be restated as

$$\Phi(\pi_1, \pi_2, \ldots, \pi_p) = 0 \ , \tag{38}$$

where the π_w are dimensionless parameters constructed from the X_w by $\phi = \nu - \kappa$ equations of the form

$$\pi_w = X_1^{\alpha 1} X_2^{\alpha 2} \ldots X_\nu^{\alpha \nu} \ , \tag{39}$$

where the exponents α_w (Greek symbol *alpha*, not to be confused with a lowercase *a*, which the author retains for the natural sequence) are rational numbers. In other words, the dimensions of (36) might be considered to be the same as those in $B^{1 \div D}$; however, it accurately amounts to a dimensionless quantity when considering the infinite function it belongs to, expressed in π_w, a dimensionless function. Thus, continue to apply the values of *k*, $f_x(w-3)$ and Mass in (19) to solve for the next. This gives

$$\begin{aligned}\pi_2 &= f_x(2)\exp\left(k + \ln\left(\left|Y_1\right| \div f_x(-1)\right)\right)\\ &= f_x(2)\exp\left(6.271\ldots + \ln\left(\left|9.109\ldots\times 10^{-31}\right| \div 2.197\ldots\times 10^{-2}\right)\right)\\ &= 5.685\ldots\times 10^{-12}\ \text{dimensionless}\ .\end{aligned} \tag{40}$$

where the absolute value of X_{w-3} equals the absolute value of Y_1, which is Mass, and so its value is used to solve for π_2.

Then apply the values *k*, $f_x(w-3)$ and either (23) or (25) to solve for the π_3, where the Null Object is now used from the single Element in Set Z_w.

$$\begin{aligned}\pi_3 &= f_x(3)\exp\left(k + \ln\left(\left|x_0, y_o\right| \div f_x(0)\right)\right)\\ &= f_x(3)\exp\left(6.271\ldots + \ln\left(\left|4.163\ldots\times 10^{-16}\right| \div 5000\right)\right)\\ &= 2.598\ldots\times 10^{3}\ \text{dimensionless}\ .\end{aligned} \tag{41}$$

where the absolute value of X_{w-3} is equal to x_o, y_0, which is some field in kg-s-1 (to be mathematically proven in this paper as the Vacuum Field), and so its value (which can be solved without knowing what field it is at this point) is used to solve for π_3.

Comparing the values then of X_w to that of the second term in the second absolute value of (13), the values of π_w, as shown in Table 4,

w	X_w	π_w
1	-2.598… E+03	-2.598… E+03
2	-2.598… E+03	2.598… E+03
3	5.991… E-12	5.991… E-12
4	-1.621… E+22	1.621… E+22
5	-1.621… E+22	1.621… E+22
6	3.355… E+07	3.689… E+07
7	-1.012… E+41	1.012… E+41
8	-1.012… E+41	1.012… E+41
9	2.127… E+26	2.094… E+26
10	-6.318… E+59	6.318… E+59

Table 4. Function Comparision of Values

One notices a number of cases where π_w is the negative counterpart of X_w, and that any values different eventually converge on zero as *w* approaches infinity.

However, this does pose an equation arrangement problem for π_w and X_w as π_w to infinity is undefined if the sum of X_w ever were to reach zero; thus far, it is only known to not generalize. In $\ln\left(\left|X_{w-3}\right| \div f_x(w-3)\right)$, when the sum of $X_{w-3}=0$, the entire equation is undefined, as $\ln\left(\left|X_{w-3}\right| \div f_x(w-3)\right)$ would reduce to $\ln(0)$. Suffice to say,

$$\lim\left(\text{Compositionof Map}\,\Theta\right)=\left|X_w\right|-\left|\pi_w\right|=0\,,\quad w\to\infty \tag{42}$$

By rearranging π_w in order to solve for the absolute value of X_{w-3}, it is easy to see how the absolute value of X_{w-3} must equal zero, as π_w clearly equals zero, as it is built upon Groups where the product of every three summed Spaces equals zero to infinity in (30). To rearrange in a manner such that the equation is defined, begin at

$$\pi_w = f_x(w)\exp\left(k+\ln\left(\left|X_{w-3}\right| \div f_x(w-3)\right)\right) \tag{43}$$

Divide both sides by $f_x(w)$, give the natural logarithm of both sides, subtract the natural logarithm of $\left|X_{w-3}\right| \div f_x(w-3)$ from both sides, which by rearranging, using the rules of logarithms that states $\log(x)-\log(y)=\log(\frac{x}{y})$, one can derive

$$\ln\left(\frac{f_x(w-3)\pi_w}{f_x(w)\left|X_{w-3}\right|}\right)=k \tag{44}$$

Lastly, give the exponent of both sides, multiple $\left|X_{w-3}\right|$ by both sides and divide both sides by the exponent of *k*. This gives

$$\frac{f_x(w-3)\pi_w}{f_x(w)\exp(k)}=\left|X_{w-3}\right| \tag{45}$$

Given in (30), (33) and (39) that $\pi_w=0$, the left side of the equation reduces to zero, where one gets,

$$0=\left|X_{w-3}\right| \tag{46}$$

which provides a defined solution for the absolute value of X_{w-3}, which can then allow mapping to the *Z* Field.

Since the difference of the infinite sums in (42) equals zero, defined in (46), an expression for the *Z* Field results in (47), as Y_w converges on the only sum greater zero.

$$\text{Continuous Map } \Theta \text{ of this sequence} \;:\; -Y_w = Z_w \tag{47}$$

Lastly, place all these Topological Spaces, including those in X_w whose sum equals zero, into the Lagrangian in the next section such that all Spaces are defined in absolute values, the completion of the proof results, the inclusion of all the Field summation functions in its definition of the theorem. The final result of the proof of the theorem:

$$Z_w=\left|Y_w\right|+\left\|X_w\right|-\left|\,\pi_w\right\|\;, \tag{48}$$

where Z_w equals the sum of all the Fields in Space, the very definition modern scientists give for the Vacuum Field *Z*; the Field on the left hand side of (48) can be nothing other than the *Z* Field by most any reasonable understanding, as Z_w only contains the sum of a single Field. Thus,

$$\begin{aligned}\text{Map}\,\Theta &= \left|Y_w\right|+\left\|X_w\right|-\left|f_x(w)\exp\left(\left(k+\ln\left(\left|X_{w-3}\right|\div f_x(w-3)\right)\right)\right\|\\ &= Z_w = Z\,\text{Field}\left[\text{kg-s}^{-1}\right]\end{aligned} \tag{49}$$

where the SI unit dimensions of the *Z* Field amount to just kg-s^{-1}.

The Universal Field Lagrangian

The composition of map Θ, which is $\left|X_w\right|-\left|\pi_w\right|$ equals zero at an infinite number of *w: w* = 1, 2, 4, 5, 7, 8, 10, etc., referring to Table 4 above, as they are modular to three. Thus, at any point θ in the composition of Θ,

$$\theta=0 \quad\Leftrightarrow\quad w \bmod 3=0 \tag{50}$$

For the Lagrangian definition, one gets

$$L=T-V: \qquad \begin{aligned} L&=Z_w\\ T&=X_w-\pi_w\\ V&=Y_w\end{aligned} \tag{51}$$

The Z Field Velocity-Action Theorem

While this theorem could encompass a separate paper, it turns out that in order to satisfy the Lagrangian proof from the last section (solving for $q(t)$), a proof of this theorem results in the author's **Field-Action Hypothesis**, validating it also as a theorem.

5. Field-Action Hypothesis

The hypothesis is that if the Action *q* of any Field Space F_N (any Element from *x* or *y: B* Field, *A* Field, *Z* Field, etc., known or unknown) is that which interacts in F_N in order to provide Force N (as mentioned earlier, this Force *N* is not to be confused with the N Field, which will be denoted with a different symbol in future publications) on a System having a given Mass, then the dimensions and value of that Action can always be calculated as the ratio

$$-q = \frac{N}{F_N} \tag{51}$$

where Action q always becomes negative as a result of this ratio, N is the measurement of Force on the System (always how much Distance a System having a given Mass can transverse in a given amount of Time and always in $kg\ m\ s^{-2}$) and F_N is any Field of Force from the Universal Field Lagrangian, from category *Grp*, containing the Elements x, y.

The reason the author thinks this is because it is already evidenced dimensionally in the B, A and E Fields, and there is no clear reason otherwise that any Field to infinity should not be calculated in the same manner (Gravity, Electroweak, Nuclear Strong, etc.). For example, in terms of the E Field in $kg\ m\ C^{-1}\ s^{-2}$, referring back to (15),(16) & (17), the Action is calculated to electric Charge C, that which interacts in the electric Field.

$$\frac{N}{E}\left[\frac{\text{Mrt}^{-2}}{\text{Mre}^{-1}\text{t}^{-2}}\right] = -e = -q_E\left[C\right] \ : \ F_N = E \ , \tag{52}$$

where minus Action q of Electric Field E is minus Electric Charge in Coulombs. Also, in terms of the A Field in kg-m-C^{-1}-s^{-1}, the Action is calculated to electric current j in C-s^{-1},

$$\frac{N}{A}\left[\frac{\text{Mrt}^{-2}}{\text{Mre}^{-1}\text{t}^{-1}}\right] = -j = -q_A\left[\text{C-s}^{-1}\right] \ : \ F_N = A \tag{53}$$

And in terms of the B Field in kg-C^{-1}-s^{-1}, the Action is calculated to be the scalar magnetic potential Ψ in C-m-s^{-1},

$$\frac{N}{B}\left[\frac{\text{Mrt}^{-2}}{\text{Me}^{-1}\text{t}^{-1}}\right] = -\Psi = -q_B\left[\text{C-m-s}^{-1}\right] \ : \ F_N = B \tag{54}$$

The hypothesis suggests that the reason the Actions in (52-54) follow suit in these commonly known Fields is because it involves some series that governs a pattern for all Fields, any Field from plus or minus infinity (Gravity, Z Field, etc.). It also implies that each Field has a unique Action that only it interacts with (the dimensions of one Action for any particular Field is always different from that of another Field).

In terms of Gravity, both Newton and Einstein (and therefore almost all other physicists) would consider (in this sense) the Action to be Mass, as indeed Newton's equation for gravitational interaction involves two Masses times G (the gravitational constant) over Time squared [20]. Considering Mass to be the Action of the Gravitational Field, Einstein would perhaps reduce the Field to dimensions of **Acceleration** g in m-s^{-2} [6]. However, above, the Universal Field Lagrangian provides the calculation as Mass being itself a Field (in $kg\ m^0\ C^0\ s^0$ or just kg), and considering the hypothesis, the Action is considered the Acceleration of the System, as calculated in (55).

$$\frac{N}{M}\left[\frac{\text{Mrt}^{-2}}{\text{M}}\right] = -\text{g} = -q_M\left[\text{m-s}^{-2}\right] \ : \ F_N = M \ , \tag{55}$$

where g is acceleration.

In terms of non-uniform gases, a topological Mass Field (as well as Time and space frames), Eq. (55) makes sense for Relativity, and Relativity should not require any differentiation (and perhaps does not) between which is the Action and which is the Field. In other words, if Relativity suggests Mass curves space and Time, affecting Acceleration, then this model would only suggest/allow the counter: Acceleration would curve space and Time, affecting Mass, not vice versa. This differentiation is important for this model, though perhaps not for Relativity.

6. Proof of the Field-Action Hypothesis

The author solves for the Action of the Z Field, whose dimensions were proven earlier to be in kilograms per second from the Lagrangian. If the Z Field's Action could be reduced to Velocity, as in (56) below, from the Lagrangian, then by mathematical consequence of the Euler-Lagrange equation, all Fields would follow suit in the hypothesis, having the System's negative Action equal to the Force divided by the Field.

$$\frac{N}{Z}\left[\frac{\text{Mrt}^{-2}}{\text{Mt}^{-1}}\right] = -v = -q_Z\left[\text{m-s}^{-1}\right] \ : \ F_N = Z \tag{56}$$

This would suggest for the Z Field that anything moving (having some Velocity) interacts with this Field, which further suggests it to be the single most fundamental (virtual) Field in nature, considering everything in the universe is moving with respect to some other System, as well as the fact that Force and the Fields involve the movement of Systems by their very definition. This could also indirectly result that the Bernoulli Principle (a principle of flow, pressure and Velocity underlying water pumps, sail boats, airplane lift, and even a curve ball thrown in baseball) is governed primarily by the Z Field.

The method for solving for the Action of a Lagrangian was discussed earlier, where several instances t of T in $L = T - V$, are calculated and graphed against L. The value of the area under the curve is then the value of the Action q. Since each instance t is a point in X, where L is the value of the Z Field of our above electron example. Since the values of such a graph extend to the far positive and far negative (as well as the infinitesimally small and large) using the instances t of space X, from the Lie Group Fields, a to-scale graph is not practical, as this curve experiences the same difficulties as before, requiring another Net. But such difficulties are still manageable for calculation, so long as the values are known and the general shape of the curve is known. If this were a smooth curve, such problems might be easier (or maybe more difficult), but in this case the graph forms the geometry of triangles and but two rectangles early on for each natural number w spacing them, always a value difference of one (2 – 1 = 1, 3 – 2 = 1, 4 – 3 = 1, etc.).

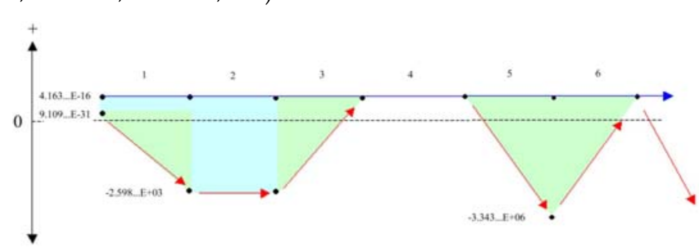

Fig. 3. Graph of $X_w - \pi_w$ against Z (not to scale)

Then calculating the area of each colored region in *Fig. 3*, and summing those areas, one gets the Action as the value of the sum, function q of t. The instances where the area equals zero (as shown at instance 4) repeat every three spaces to infinity, which hints at something modular in its composition, as will be utilized in the coming equations, and also why the sum does not increase between instance three and instance four, as well as every other three values of w. The values of this function for the first six instances are shown in Table 5.

w	$q(t)$
1	1299.31…
2	3896.93…
3	5197.24…
4	5197.24…
5	1676995.98…
6	3348794.73…

Table 5. First Six Values of $q(t)$

Graphing this function is a bit cumbersome, as the values far exceed the proceeding, so the author presents a graph for the natural logarithm of the function in *Fig. 4* for better illustration.

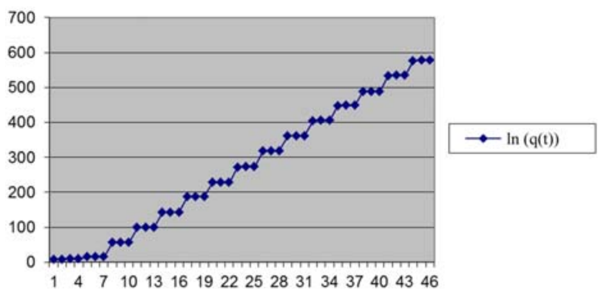

Fig. 4. Graph of the Natural Logarithm of $q_Z(t)$

The Proof

The author will prove that the natural logarithm of $q_z(t)$ for the Z Field minus velocity function $v(w)$ is $O(3)$, thus resulting in the dimensions meters per second.

$$\ln(q_Z(t)) - v(w) = O(3) \tag{57}$$

By means of the theorems governing big oh definitions [18], if any difference between the three-modular function of the Z Field's Action and a three-modular Velocity is always less than a dimensionless three, then the dimensions for the Action inevitably equals the dimensions of Velocity.

In an arbitrary, but directed mathematical construct, the modular aspect of v of w can be based on a modular algorithm function $\beta\ (w)$ involving the three Fields for each unitary group to infinity in the following way. Given the smooth manifold of the electron (a conserved System, as we are considering instances t to involve the Lie Group Fields, which exists in Systems having smooth manifolds) having the same parameters as the earlier example (Charge, Mass, Distance, Time, etc.), let each Field have a propagation Velocity measured from outside the electron's radius, a repetitive Action caused by the same propagating wave (example used earlier), which travels at –1 $m\ s^{-1}$. This would be analogous to how long an inductor in an electrical circuit takes to form a magnetic Field depending on its size and permeability of its core, or for how long it takes to build an electric Field in a capacitor, depending on its size and dielectric material; the strength of the Fields at certain Distances change over Time when forming or decaying, which can be described as a Field propagation in terms of Velocity. Again, considering a wave propagating at –1 m-s^{-1} inside the radius of the electron, the energies of all Fields in such a System become equal. This does not necessarily say that the electron has some sort of ad hoc external quantum shells, nor does it suggest that all Fields are naturally finite in Distance (like the nuclear strong Field); rather, it just assumes that all Lie Group Fields could be limited/shielded to some Distance (Gravity is not a Lie Group Field, for those who may question if Gravity can be shielded enough to assume the following equations) using some hypothetical method in order that one may calculate in the following manner. Since there is no mechanism defined by physics that requires an electron to consist of such Field boundaries, let our smooth manifold System act as an artificial electron, having the same values of Charge, Mass, radius and the like, but one that implements this limiting mechanism without affecting its other parameters.

Considering outside Observer A, measuring the relative Velocity of the repetitive Action's Field propagation velocities and the wave propagation Velocity (the wave that induces the Field propagation velocities), due to Heisenberg's Uncertainty Principle, Observer A cannot measure both the location of the Action and the momentum of the Action at the same Time, as they are mutually complimentary. Thus, because the Distance in meters of the Velocity in meters per second is linked to the location of the Action and the Time in seconds is linked to the momentum of the Action, the operators representing the variables Distance and Time do not commute. However, since the Distance and Time values are both negative counterparts to one another for the Velocity of this hypothetical wave, one can choose either measurement and calculate the other, one of Distance or one of Time. The consequence however remains that a single unit of one can only be used to describe the other for any accurate outside observer measurement (Time = 1 second or Distance = 1 meter).

Thus considering all such restrictions, let the velocity algorithm $\beta(w)$ for his or her repetitive Action velocity observation be defined as follows, since all Actions are repetitive, otherwise they would be better described as "flow", the frequency, inverse of Time is a better choice for this measurement. Let

$$\begin{array}{lr} \beta(w) = \text{if}(w \bmod 3 = 0) & \frac{1}{2}\ln v_f + 2v_p \\ \quad \text{else if}(w \bmod 3 = 1) & \ln v_f + 3v_p \\ \quad \text{else} & -2v_p\ , \end{array} \tag{58}$$

where

$$v_f \equiv 1 \text{ meter} \times \text{wave Frequency}$$
$$v_p \equiv \text{wave propagation Velocity}$$

This algorithm uses additive Velocities instead of the Lorentz Factor when considering relative Velocities in order to avoid complex values that would occur with the Lorentz Factor. Relativity uses the Lorentz Factor instead of additive velocities and avoids imaginary figures by considering the Velocity of light to be a limiting Velocity for moving bodies. In the event that Relativity would require the Lorentz Factor in terms of this derivation instead of Classical Mechanics' additive Velocities, the author reminds that this proof only is intended to solve mathematically for the dimensional units of the *Z* Field and is not limited to any relativistic conventions in this regard.

Elucidation of the First Conditional

When *w* mod three equals zero (an arbitrary, but directed modular condition, allowable considering the modular Field series), then the relative Velocity occurs at β of *w* equal to the natural logarithm of one meter times the propagation frequency (1 / wave Time), divided by two, plus two times the wave propagation Velocity. Since the wave propagation Velocity equals minus one, the result is simply the first term minus the number two. The number two is only a magnitude relation between the Field propagation Velocity and the wave propagation Velocity—and again, arbitrary but directed.

Elucidation of the Second Conditional

When *w* mod three equals one (another arbitrary, but directed modular condition, allowable considering the modular Field series), then the relative Velocity occurs at β of *w* equal to the natural logarithm of one meter times the propagation frequency, plus three times the wave propagation Velocity. Similar to the first conditional, since the wave propagation Velocity equals minus one, the result is simply the first term minus the number three. In the same, the number three is only a magnitude relation between the Field propagation Velocity and the wave propagation Velocity—arbitrary but directed.

Elucidation of the Third Conditional

When *w* mod three equals two (the final arbitrary, but directed modular condition, allowable considering the modular Field series), then the relative Velocity occurs at β of *w* equal to a minus two times the wave propagation Velocity. In other words, this value always equals just the number two, as the wave propagation Velocity equals minus one. The magnitude is a negative integer only in order to result in a positive relative Velocity value, as the measurement takes place within the same radius as Observer *A*); the magnitude is actually just two times that of the wave propagation Velocity.

Using the value of the Electron Radius from earlier, applying it to this algorithm, one gets

$$\beta(w) = 30.775\ldots, 2, 14.877\ldots, 30.775\ldots, 2, 14.877\ldots, 30.775\ldots \quad (59)$$

Let another Velocity function *h* of *w* (not algorithmic like β of *w*) of a repetitive Action with respect to a unit meter and a fixed Frequency (the inverse of the same wave Time) that is equal to a minus Electron Radius, still a Velocity function, be unobservable to Observer *A*.

$$h(w) = w + \frac{w}{2} - \ln(1 \text{ meter} \times \text{wave Frequency}) + 1\frac{1}{2} \quad (60)$$

This could be described as an Action taking place in a negative counterpart space, as *h* of *w* is always negative and β of *w* is always positive to infinity. This of course is a purely hypothetical situation, but it does provide a mathematical framework in order to prove what is observable in first hand experiments. Then let Observer *B* observe Observer *A*'s Actions (his repetitive measurements) and the modular Velocity Action from function β of *w*, who (unlike Observer *A*), can indeed observe Action *h* of *w*, which again, by convention, places this System within another spatial radius (returning the negative value). This gives the relative Velocity of an Action within an Action, which is

$$u(h(w), \beta(w)) = h(w) - \beta(w) \ . \quad (61)$$

Observer *B*'s consideration of Lie Groups measurements cannot easily be calculated with the Topological Groups in order to solve for the *Z* Field's Action with another Net, and a dimensionless function $f_u(w)$, in the domain of natural *w*:

$$f_u(w) = w - \frac{14\ (w-1)}{1+w^{-1}} \ . \quad (62)$$

Thus,

$$v_u(w) = u(h(w), \beta(w)) - f_u(w) \ . \quad (63)$$

By subtracting $v_u(w)$ from $q(t)$ for all *t* from one to infinity, one gets the result of the proof, nearly (will be refined) a restatement (57):

$$\ln(q_Z(t)) - v_u(w) = O(3) \ . \quad (64)$$

Fig. 5 better illustrates these bounds.

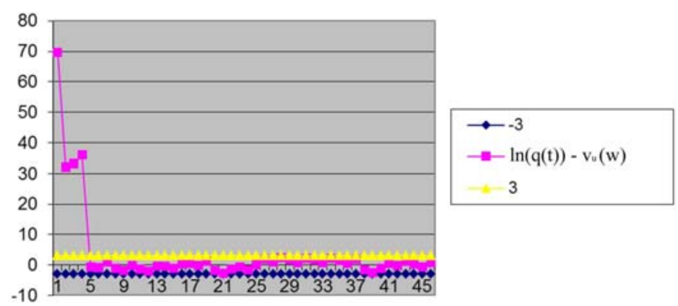

Fig. 5. $\ln(q_Z(t)) - v_u(w) = O(3)$

And Fig. 6 best illustrates their equivalence to infinity.

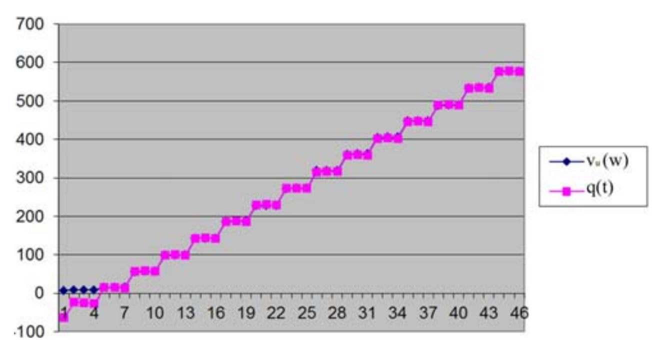

Fig. 6. Graph of $q_Z(t)$ and $v_u(w)$.

$$q_Z(t) \sim \exp(v_u(w)) \ , \quad (65)$$

or more precisely,

$$q_Z(t) = \exp(v_u(w)) + \varepsilon_u \ , \quad (66)$$

where ε_u is some error term.

7. Reduction of the Error Term

Because this function is modular, the error term is also modular and can be calculated to further accuracy, particularly reducing the earliest values. Given Euler and Gauss' study in this area of mathematics [15],

$$\rho_0 = (m_o \bmod d) \equiv 1, \rho_1 = (m_1 \bmod d) \equiv 2, \rho_3 = (m_3 \bmod d) \equiv 3, \ldots \quad (67)$$

Where mathematicians attempt to determine if arbitrary γ is divisible by some divisor d, also an integer, an infinite Sequence arises: $R(d)_m = (\rho_0, \rho_1, \rho_2, \rho_3, \ldots)$, the residue Sequence for divisor d base m. Any such periodic series of values ρ_w can be infinitely defined as follows:

$$\eta(m,w) = \frac{m_1 - \rho_1}{d}, \frac{m_2 - \rho_2}{d}, \frac{m_3 - \rho_3}{d}, \ldots = 0 = (\varsigma(w))^w , \quad (68)$$

which begins at some given magnitude (though is not first countable) and always converges to zero, where $\rho = m \bmod d$ of which arbitrary γ is divisible by d.

Since all the error terms of (66) eventually have no magnitude greater than the absolute value of three, one can add three to both sides of (66) in order to always solve for a positive ε, which helps in the reduction of the term. Thus, taking a look at the error terms of (66), one gets the largest possible value of $\varepsilon + 3 = 72.681\ldots$ at $w = 1$ and the rest quickly drop below six. The residue for this series then is less than six, thus d can equal six as it is always greater than ρ. By applying the error term plus three to (66) with respect to u, one gets convergence even as earlier as $w = 5$, where $\eta(5) = 0$.

$$\eta_u(m,w) = \frac{72.68\ldots - 0.68\ldots}{6}, \frac{35.19\ldots - 5.19\ldots}{6}, \ldots \quad (69)$$

$$\eta_u(m,w) = 12,5,6,6,0,0,0,\ldots \quad (70)$$

Next, $\varsigma(w)$ from (68) with respect to u can then be solved using (71) below, in order to satisfy the logarithmic nature of $\eta_u(m,w)$. One gets

$$\varsigma_u(w) = \exp\left(\ln(\eta_u(m,w)) \div w\right) , \quad (71)$$

which also converges at w equals five, but to Complex values.

Considering (64), where the natural logarithm of q of t minus $v_u(w)$ is $O(3)$, but considering the modular natures of these functions, a further reduction of the error term of (66) can be made in the same manner as (67) through (71), but the exact derivation is not shown, but it was performed using the identical method for the first value of m, but with a varied completed rearrangement of the terms. The completed more precise result is presented in (72), which results simply in a more precise Net.

$$v(w) = u(h(w), \beta(w)) - f_u(w) - w + 6\eta_u(m,n) - \frac{1}{2} \quad (72)$$

$$\ln(q_Z(t)) - v(w) = O(3) , \quad (73)$$

which is the completion of the proof of this theorem, the Action (that which interacts) of the Z Field is *Velocity* itself, as the dimensions of the exponent of meters per second is still measured in meters per second.

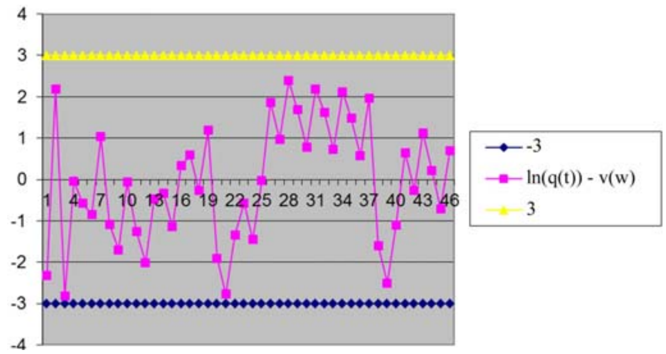

Fig. 7. Graph of $\ln q_Z(t) - v(w)$.

This is far better bound than that shown in Fig. 5, as it bounds all values beginning from w equals one. One also sees a closer equivalence between the natural logarithm of $q_Z(t)$ and $v(w)$ than that in Fig. 6.

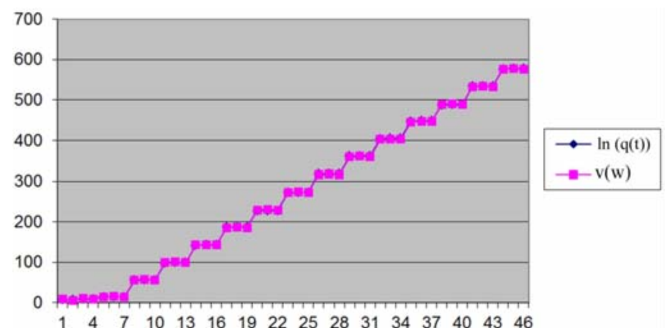

Fig. 8. Graph of $q_Z(t)$ and $v(w)$

Thus, one can state the Action of the Z Field (for this proof) as follows:

$$q_Z(t) = \exp(v(w)) + \varepsilon\left[\text{ m-s}^{-1}\right] . \quad (74)$$

Since the error term ε is represented in the $O(3)$ (a natural number) side of the equation in (57) and everything to the right of $u(h(w), \beta(w))$ in (72) are either natural numbers or functions in the domain of natural numbers, everything in the Action of the Z Field reduces to dimensionless quantities with the one exception of the first term, which is in meters per second. This supports the Field-Action Hypothesis and satisfies (38). While the Z Field's value is positive and the Action is positive, the Force must be negative when considering the two. Negative Forces and energies are not uncommon and used when calculating Gravity and the Force in the launching of rockets into space [23]. Additionally, with that said, the dimensions are the most important aspect of this study. Because of the nature of a Lagrangian, particularly the one in this paper, not only is the Field-Action Hypothesis valid for the *B* Field, the *A* Field and the *E* Field, but also because it is valid for the Z Field (the sum of all Fields in space, not a System's ground state Field), it must be valid for all conceivable Fields.

Field-Action Theorem:

$$\text{Force} = \text{Field of Force} \times \text{minus Action} \quad (75)$$

8. Conclusion

The above derivations not only reduce the sum of all Fields known or unknown to dimensions in kilograms per second, but also that those are the physical dimensions of the Z Field. Inversely, it proves that because such a mathematical derivation can be performed in the form of a Lagrangian, there must be by consequence an infinite possible Fields in the universe (a far stretch from just the four known Forces discovered by scientists today) as well as an infinite number unitary groups, and the above Lagrangian unifies them in a single equation. While one may suspect that some of these infinite number of possible Fields may be mathematical equivalents of another (as rightfully they may be, though not hypothesized in this paper), each Field that could be described as unique to another would be required to involve a separately unique Action, as was fulfilled above in the proof of the Field-Action Theorem; in other words, all the Fields in the Lagrangian may not be unique (there may be repetitions), but there still is an infinite number of unique Fields within the Lagrangian. While mathematically the Action a Field interacts with in the Topological Group may be reduced to a different Action for a different Field in the Lie Group (for instance, a hypothetical "Electrogravitic Field" that behaves similarly or opposite to the more commonly understood Gravitic Field in the Topological Group), this could only occur if 1) the Actions involving the two Fields are somehow mathematically equivalent, which would result in mathematically equivalent Field dimensions or 2) such would arise by mere coincidence, which could certainly be expected considering an infinite number of Fields and an infinite number of Actions, where such coincidences may certainly be common.

Because this paper involved multiple disciplines of mathematics (clock mathematics, calculus, topology, analytical number theory and more) that are tied strongly to Gauge Theory in physics (yet another discipline), it is understandable that peer review on such matters should take years for acceptance, as such disciplines typically do not always carry over easily. With that said, it was not this author's intent for timely acceptance; rather, a means to document the mathematics discovered that has already been backed up with first hand, reproducible experiments. The true intent of this paper was to serve as a theoretical road map for future experimentation and for thorough understandings, as well as reference to workable equations above that will be used for said future experimentation of the author—and those who might consider it a useful reference tool.

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