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Demystification of the spacetime model of relativity

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Abstract: The geometrical interpretation of gravitation in general theory of relativity imparts certain mystical properties to the spacetime continuum. The mystic connotations associated with this space-time model may be attributed to the fallacious depiction of space-time as a physical entity. This paper proves that the spacetime continuum in GR is a simple mathematical model and not a physical entity.

I. NOTIONS OF SPACE AND TIME

A. Introduction

The geometrical interpretation of gravitation in General theory of Relativity (GR) implies the spacetime continuum to be a physical entity which can even be deformed and curved. This misconception is quite deep rooted in the metaphysical eternalist viewpoint of existence in contrast to the logical presentist viewpoint. As per the eternalist viewpoint, a so-called material object in a spacetime world is a continuous series of spacetime events, each of which exists eternally as a distinct part of the world. There is no distinction between the past, present and future. This is a block view of spacetime, in which the universe pre-exists at all future instants of time. As per the presentist viewpoint, the present moment is different from the past and future and that physical entities exist only in the present. The physical phenomenon does not exist in the past and the future regions of time. The foundations of GR are critically dependent on the integrity of the notion of spacetime as a physical entity. Albert Einstein had asserted in a 'matter of fact' way, "*the world in which we live is a four-dimensional space-time continuum.*"[1] According to GR, "mass curves space-time, and space-time tells the mass how to move."[2] In an interesting article, 'Cosmological Constant, Space-Time, and Physical Reality', W. Schommers, has made a significant observation, "Spacetime is not a container for the integration of physically real objects, etc., nothing can be inside spacetime—no matter, no energy. Spacetime exclusively play the role of auxiliary elements for representing the picture of reality; spacetime can only contain geometrical positions, trajectories, etc., such as a sheet of paper."[2]

Our dynamic universe is embedded in a three-dimensional (3D) Euclidean space and its dynamic behavior or characteristic changes can be represented with an independent time coordinate. Before considering the combined or interdependent features of space and time as a whole, we need to first examine different notions of space and time separately. Specifically we need to distinguish between the mathematical abstract notion of coordinate space and the physical notion of space as the container of physical objects, as the physical void in between an ensemble of material particles.

B. Coordinate Space

On a coordinate line OX, if we define line segment OA as the unit length, then length of a line segment OP can be defined as x times OA, where x=OP/OA. The association of the set of points P on coordinate line X with the set of real numbers x, constitutes a coordinate system of one-dimensional space, once the notion of certain unit length has been defined. The one-to-one correspondence of ordered pairs of numbers with the set of points in the plane X^1X^2 is the coordinate system of 2D space. Similarly, with a predefined notion of unit length, an essential feature of 3D coordinate space is the concept of one-to-one correspondence of points in space with the ordered triplet of numbers. The predefined notion of unit length or scale for different coordinate axes, constitutes the metric of space for quantifying the notion of distance and position measurements of the sets of points in this coordinate space.

We define a space (or manifold) of N dimensions as any set of objects that can be placed in a one-to-one correspondence with the ordered sets of N numbers $x^1, x^2,..., x^N$. Any particular one-to-one association of the points with the ordered sets of numbers is called a coordinate system and the numbers $x^1, x^2, ..., x^N$ are termed the coordinates of points. In all coordinate spaces that are metrized, we associate the notion of unit length along all coordinate axes and metric tensor components g_{ij} with each coordinate system. All essential *metric* properties of a metrized space are completely determined by this metric tensor.[3] However, it is pertinent to note here that at any given point P in space, it is not possible to physically measure the metric tensor components or metric coefficients. We cannot even 'define' the metric coefficients at point P without first defining the 'corresponding' coordinate system.

C. Physical Space

The notion of physical space implies the spatial extension of the universe wherein all material particles and all fields are embedded or contained. The true void between

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material points is in essence the physical space, or empty space, or free space. It is important to note here that the coordinate space, along with its unit scale or metric, is our 'human' creation intended to facilitate the quantification of relative positions of material particles and fields. The existence of physical space does not depend in any way on the existence or nonexistence of coordinate systems and coordinate spaces. Of course, for the study and analysis of physical space and the material particles and fields embedded in it, we do need the structure of coordinate systems and coordinate spaces as a quantification tool. The most significant point to be highlighted here is that whereas the metric scaling property is only associated with coordinate space, the physical properties of permittivity, permeability and intrinsic impedance are associated with physical space. The notion of material particles and fields being embedded or contained in the physical space, is generally accepted as valid. However, the detailed mechanism involved in this embedding is not known. Obviously, such a mechanism must involve the known physical properties of free space.[4]

Fundamental known properties of this physical space or free space are represented by the following dimensional parameters.

Permittivity of free space, $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2/N.\ m^2$

Permeability of free space, $\mu_0 = 1.257 \times 10^{-6}\ N / A^2$

The speed of propagation of EM waves in vacuum,

$$c = \sqrt{1/(\mu_0 \varepsilon_0)} = 2.998 \times 10^8 \text{ m/s}$$

The intrinsic impedance of vacuum,

$$Z_0 = \sqrt{\mu_0/\varepsilon_0} = 376.7 \text{ (N.m/C)/A} = 376.7 \text{ V/A} = 376.7 \text{ Ohms}$$

These four parameters, as dimensional constants, represent fundamental physical properties of vacuum or physical space. The speed 'c' of propagation of electromagnetic disturbances is governed by the permittivity ε_0 and permeability μ_0 constants associated with the physical space or vacuum. Since these four parameters are interrelated, only two of them are independent. It is interesting to note that μ_0 can be replaced with Z_0/c and $1/\varepsilon_0$ can be replaced with $c.Z_0$ in all relations involving μ_0 or ε_0. These parameters are quite routinely measured experimentally and are universally well known. It may be emphasized here that these physical properties are not correlated with the metric tensor of the coordinate space and hence cannot represent the metric properties of the coordinate space.

D. Notion of Time

Since our universe is inherently dynamic, there are a large number of physical processes in Nature which undergo cyclic changes. The notion of time is associated with relative measurement of such changes. Depending on the consistency of such cyclic changes and the convenience of their measurement, we may select any one of them as our reference scale for relative measurement of change. The angular position of a planet in orbit, the position of a pendulum oscillating about a mean and the vibrations of many electro-mechanical systems are all examples of physical processes that undergo cyclic changes. Any such cyclic process could be adopted as a reference scale for measurement of change or the reference scale for time. In general, the study of natural phenomena invariably involves the comparative study of various changes. For this comparative study, we need to use a reference scale, or more correctly a reference time scale, for relative measurement of change. Hence time, as a relative measure of change, is an important parameter in the study of an essentially dynamic physical Universe. Existence of a uniform reference time scale can thus be attributed to the consistency of the physical cyclic process adopted for the reference scale.[5]

II. GEOMETRICAL REPRESENTATION OF A RIGID 3D CONTINUUM

A. Invariance of Arc Element ds

Let us consider a 3D continuum of space points representing the points of an ideal rigid material medium. All points in this space will be considered as invariant. A point P is determined by a set of coordinates x^i in a given reference frame. If the coordinate system is changed, the point P is described by a new set of coordinates y^i, but the transformation of coordinates does nothing to the point itself, which remains invariant. A set of points, such as those forming a curve, is also invariant. The curve is described in a given coordinate system by an equation which usually changes its form when the coordinates are changed, but the curve itself remains unaltered, invariant. Similarly, a triply infinite set of points, constituting a rigid 3D space continuum, may also be considered invariant if an infinitesimal separation distance ds between any pair of neighboring points remains invariant under admissible coordinate transformations. The notion of invariance of the arc element ds in all admissible coordinate transformations is most crucial in the representation of a rigid 3D continuum. Since representation of vectors and tensors in the Euclidean geometry rely on the invariance of arc element ds, it implies that the Euclidean 3D space is effectively treated as a rigid 3D space continuum. This invariance of an arc element ds, is given by,

$$(ds)^2 = g_{ij}(x)dx^i dx^j = g_{\alpha\beta}(y)dy^\alpha dy^\beta \tag{1}$$

where $g_{ij}(x)$ are the metric tensor components in X coordinate system and $g_{\alpha\beta}(y)$ are the metric tensor components in Y coordinate system. In an orthogonal coordinate systems, the value of a metric coefficients g_{ii} determines the magnitude of corresponding base vector $\mathbf{a_i}$ as, $a_i = \sqrt{g_{ii}}$.

B. Coordinate base vectors and metric coefficients

Whereas a rigid 3D space continuum or the Euclidean space is essentially characterize by the invariance of an arc element ds in all admissible coordinate transformations, the

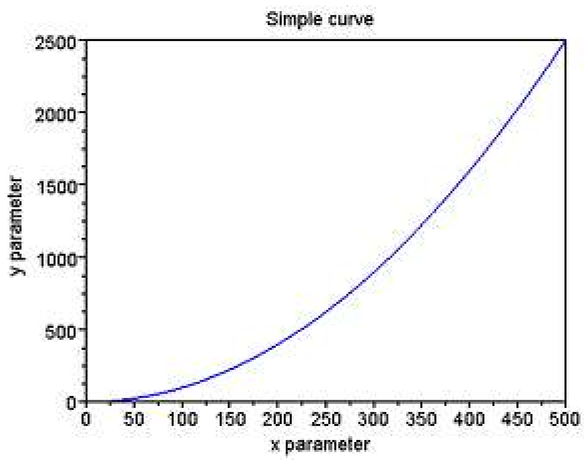

FIG. 1. Geometrical representation of a plane curve $y=a\,x^b$ on a uniform scale graph.

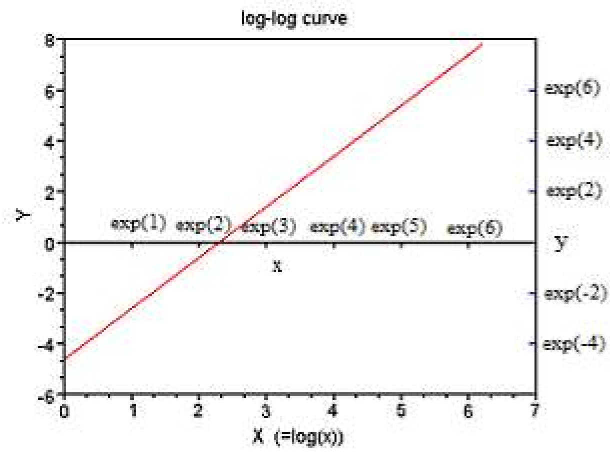

FIG. 2. Geometrical representation of a plane curve $y=a\,x^b$ on a log-log scale graph.

specific coordinate system in use is essentially characterized by the metric coefficients g_{ij}. For example, the non-zero metric coefficients $g_{11}=1$, $g_{22}=1$, $g_{33}=1$, characterize a rectangular Cartesian coordinate system (X,Y,Z) and $g_{11}=1$, $g_{22}=r^2$, $g_{33}=r^2\sin^2\theta$, characterize a spherical polar coordinate system (r,θ,ϕ). Further, as mentioned above, the individual metric coefficients in an orthogonal coordinate system, specify the unit scale or base vector of the corresponding coordinate axis. To illustrate the influence of metric coefficients g_{ij} on the *geometrical representation* of space curves, let us consider a plane curve defined by equation,

$$y = a\,x^b\ . \tag{2}$$

An x-y plot of this curve is shown in Fig. 1. Assuming a unit scale for both x and y coordinate axes, the arc element ds for this curve will be given by

$$(ds)^2 = (dx)^2 + (dy)^2\,, \tag{3}$$

which implies that the metric coefficients g_{xx} and g_{yy} are both unity for this coordinate plane. Now if we plot the curve of Eq. (2) on a log-log scale graph, the curve will take the form of a straight line, as shown at Fig. 2. Taking natural logarithm on both sides of Eq. (2), we get,

$$\log(y) = \log(a) + b.\log(x) \tag{4}$$

Let us make following substitutions in Eq. (4):

$$\log(y) = Y\ ;\ \ \log(x) = X\ ; \text{and}\ \ \log(a) = A. \tag{5}$$

Equation (4) will now take the form,

$$Y = A + b\,X \tag{6}$$

Taking differentials of Y and X,

$$dY = dy/y \quad \text{or} \quad dy = ydY = e^Y dY \tag{7}$$

and

$$dX = dx/x \quad \text{or} \quad dx = xdX = e^X dX \tag{8}$$

Substituting values of dy and dx from Eq. (7) and (8) in Eq. (3), we get

$$(ds)^2 = e^{2X}(dX)^2 + e^{2Y}(dY)^2 = g_{XX}(dX)^2 + g_{YY}(dY)^2 \tag{9}$$

This shows that the modified metric coefficients g_{XX} and g_{YY} for the X-Y coordinate space are given by exp(2X) and exp(2Y) respectively. Hence the differential scale factors or the base vectors will be exp(X) along the X coordinate and exp(Y) along the Y coordinate. The magnitude of this differential scale or the base vectors has also been shown in figure 2 along the X and Y coordinate axes. This illustrates the fact that any single-valued open curve can be represented as a straight line in a suitable coordinate system with appropriate differential scale or the base vectors. Of course, the straightening of arbitrary curves by selection of appropriate differential scales along coordinate axes is not the main issue here. The point to be stressed here is that any change in the differential scale along different coordinate axes results in corresponding change in the shape of geometric curves represented with those coordinates. Therefore, we must not view the metric coefficients in any coordinate system, as some physical entities which could physically influence or change the shape of specified open curves in the Euclidean space. It is only the *geometrical representation* of such curves in a particular coordinate system that could get influenced by any change in differential scale or metric coefficients of that system.

III. GEOMETRICAL REPRESENTATION OF A DEFORMABLE 3D CONTINUUM

A. Metric representation of continuum deformation

Let us consider a 3D continuum of space points representing the points of a deformable continuous material medium. The points in this space cannot be considered as invariant, but could be subjected to certain finite displacements without creating any discontinuity. Let us use a coordinate reference frame X for quantifying or defining the relative positions and displacements of points in this deformable space continuum. To begin with let us consider a point $P(x^1, x^2, x^3)$ in the un-deformed state of the continuum. Let $\mathbf{r}(x^1, x^2, x^3)$ be the position vector of point P. Let

Q be a point in the neighborhood of P so that the vector from P to Q, written as d**r**, can be represented in the form,

$$dr = a_i dx^i, \tag{10}$$

where a_i are the base vectors. Square of the arc element ds in the un-deformed state is given by,

$$(ds)^2 = d\mathbf{r}.d\mathbf{r} = a_i a_j dx^i dx^j = g_{ij} dx^i dx^j, \tag{11}$$

where $g_{ij}=a_i.a_j$ are metric coefficients in the un-deformed state of the continuum.

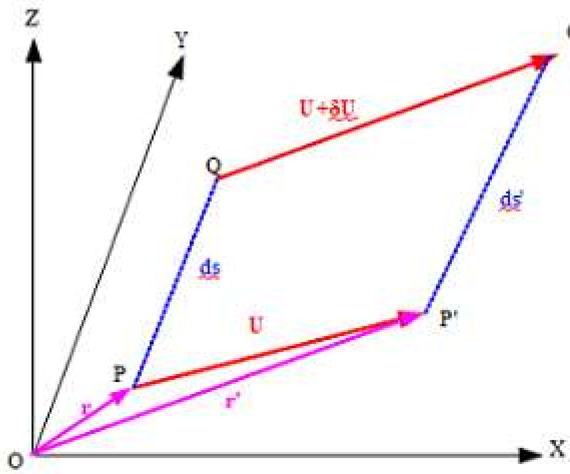

FIG. 3. Representation of displacement vector **U** in a deformable 3D continuum.

Now let us consider the deformed state of the continuum. In this state let the point P from the initial un-deformed state get shifted to point P' and the neighborhood point Q shifted to the point Q'. Let r' be the position vector of point P', as illustrated in Fig. 3. This shift in position of neighborhood points P and Q to the positions P' and Q' is termed as displacement of these points and essentially constitutes the deformation of the continuum under consideration. Let us assume that rigid body motion (i.e. translation and rotation as a whole) of the continuum is not possible and all displacements of points constitute pure deformation of the continuum. In the deformed state, the vector from point P' to Q' written as dr' can be represented in the form,

$$d\mathbf{r}' = b_i dx^i, \tag{12}$$

where, b_i are the base vectors. Square of the arc element ds' in the deformed state is given by

$$(ds')^2 = d\mathbf{r}'.d\mathbf{r}' = b_i b_j dx^i dx^j = h_{ij} dx^i dx^j, \tag{13}$$

where, $h_{ij}=b_i.b_j$ are metric coefficients in the deformed state of the continuum.

The displacement of point P to P' is represented by a displacement vector **U** and the corresponding displacement of its neighborhood point Q to Q' is given by the incremented displacement vector **U**+d**U**. The complete deformation of the continuum can be said to be fully determined when the displacement of every point P in the continuum is known or uniquely determined. The existence of displacement vector **U** at every point P, as a function of position coordinates, will constitute a displacement vector field **U** in the continuum. The displacement vector from point P to P' is given by the relation,

$$\mathbf{U} = \mathbf{r}' - \mathbf{r} = a_i u^i, \tag{14}$$

where u^i are the contravariant components of vector **U**. Differentiating Eq. (14) we get,

$$\frac{\partial \mathbf{U}}{\partial x^i} = \frac{\partial \mathbf{r}'}{\partial x^i} - \frac{\partial \mathbf{r}}{\partial x^i} = b_i - a_i \tag{15}$$

Or, $$b_i = a_i + \partial\mathbf{U}/\partial x^i \tag{16}$$

An infinitesimal deformed state of the continuum can be described as its strained state. The strained state is represented by a strain tensor E with its components e_{ij} defined at every point P of the continuum. In the linear or infinitesimal theory of deformation, the strain tensor components are computed from the covariant derivatives of the displacement vector. However, the strained state of the continuum can also be represented by the metric h_{ij} of the deformed sate. We can say that the deformable continuum is strained whenever arc element ds' given by Eq. (13) is different from the arc element ds given by Eq. (11). The covariant strain tensor components e_{ij} are related to this difference through following relations.[3]

$$(ds')^2 - (ds)^2 = (h_{ij} - g_{ij}) dx^i dx^j = 2e_{ij} dx^i dx^j, \tag{17}$$

where $$2e_{ij} = h_{ij} - g_{ij} = b_i b_j - a_i a_j \tag{18}$$

Ideally speaking, we should be in a position to obtain the displacement vector field **U** for the strained state of the continuum and then compute the components of the strain tensor and the modified metric. However, on physical considerations we may fix or specify the components of the strain tensor or the modified metric first and then work out the displacement vector field **U**. Physical constraints demand that the displacement vector field components must be finite, continuous, single valued and piecewise smooth functions of coordinates.

B. Deformable Riemannian 3-D space

The GR is based on Riemannian 3D space in which the points of the space continuum are not considered invariant. In GR, the coefficients of metric tensor $[h_{ij}]$ are obtained from Einstein's Field Equations (EFE) and the Riemann '*curvature*' tensor R^i_{jkl} computed from h_{ij} is non-zero. On the other hand, the Riemann tensor computed from the metric tensor $[g_{ij}]$ of the Euclidean space, is always zero. As such the Riemannian 3D space of GR is *defined* to be a deformable space which is generally *perceived* as 'curved' space. The Space-time continuum of the General Theory of Relativity is not a Euclidean Continuum.[1,6] Obviously the Euclidean and Riemannian geometries cannot be transformed into one another through admissible coordinate transformations. When a surface is represented in the parametric form by 2D surface coordinates, the intrinsic geometry of the surface is described by its 2D metric tensor. The Riemann tensor composed from the 2D metric coefficients is non-zero for a curved surface and zero for a plane surface. Let us critically examine the process under which a plane surface with Euclidean geometry can be changed over to a curved surface with Riemannian geometry.

Consider a large circular metal ring of radius R, filled inside with a plane thin soap-film membrane. The intrinsic geometry of any small region of this film can be represented by a 2D flat metric with zero Riemann tensor. Let us now exert a steady pressure over a small localized region of this film by impinging an air jet in such a way that a small hemispherical bubble of radius $r \ll R$ is formed in this local region. The 2D surface of this hemispherical bubble can be represented by a modified 2D metric with non-zero Riemann tensor. Obviously, it is not difficult to visualize that the localized hemispherical bubble induced by a steady external pressure is actually a deformed (elongated/stretched) membrane with a curved surface in comparison to the un-deformed plane membrane in the surrounding region. By moving the impinging air jet sideways, the location of the hemispherical bubble on the large plane membrane can be easily shifted. The state of deformation of the curved membrane in comparison to the plane membrane can be studied in detail by comparing the Riemannian metric of the curved surface with the Euclidean metric of the plane surface. The essential point to be stressed here is that a plane membrane surface with Euclidean metric *does get deformed* into a curved surface with Riemannian metric under the influence of external pressure. Precisely in the same way it has been *postulated* in GR that "flat" space with Euclidean metric *gets deformed* to a 'curved' space with Riemannian metric under the influence of a steady state gravitational field.[4]

IV. GEOMETRICAL REPRESENTATION OF DYNAMIC PHENOMENA AND TEMPORAL CHANGES

Let us monitor some dynamic phenomenon over a period of time to study the changes occurring in that phenomenon and their causal connections if any. For that we need to take instant to instant 'snapshots' of all relevant physical measurements over a period of time and then analyze the temporal changes in each of the relevant physical parameters.[7] Most common and convenient method to study such temporal changes is to make a geometrical representation of such changes through graphical plots of relevant parameters with respect to time. Let us illustrate this approach with some simple examples.

A. Geometrical Representation in XYT Coordinate Space

Consider a particle motion along the X-coordinate. This motion can be represented through a distance-time curve or trace of distance-time data points (x_1,t_1; x_2,t_2; …x_i,t_i) on an X-T coordinate plane. The velocity and acceleration of the particle at any point along the X-axis will be represented by the slope and curvature of the trace at that point. Let us now consider a particle moving in a circular orbit in XY plane. The motion of this particle can be represented as a helical trace in a XY-T coordinate space. The velocity and acceleration characteristics of this particle will be represented by the geometry of helical trace in the XY-T coordinate space. An important point to be noted here is that the helical trace does not physically exist anywhere at any time; it is just a mathematical or graphical representation of the motion of a particle over a period of time. Another important feature of this graphical representation of distance-time data points is that the X-T or XY-T coordinate space is not metrized like the Euclidean space to ensure the invariance of space points [Eq. 1]. Here the distance and time *scales*, along their respective coordinates, can be fixed independently without any constraint on the invariance of arc element ds. That is, the metric coefficients g_{ij} of this XY-T coordinate space or manifold are not constrained by the invariance of arc element ds [Eq. 1]. What is invariant in this case is the data point set (x_i,t_i) which is represented by the plot or trace on the distance-time coordinate space. The shape of this data curve can be varied arbitrarily by adjusting the individual coordinate scale or the metric coefficients of the distance and time coordinates independently. Hence, this data curve cannot be regarded as invariant under admissible coordinate transformations. In fact the notion of "admissible" coordinate transformations itself cannot be valid without some invariance constraint on the arc element ds.

However, it is possible to introduce an important constraint on the metric coefficients of the distance and time coordinates, on the lines of Minkowski space-time manifold as follows:

$$(dS)^2 = g_{tt}(c.dt)^2 - \left\{g_{xx}(dx)^2 + g_{yy}(dy)^2 + g_{zz}(dz)^2\right\}, \quad (19)$$

where dS is an invariant; g_{tt} is the metric coefficient of the time coordinate; and g_{xx}, g_{yy}, and g_{zz} are the metric coefficients of the X, Y and Z coordinates respectively. With the introduction of this constraint on metric coefficients, the distance and time scales get interlinked such that the trajectory trace of the data point set (x_i,t_i) in the distance-time coordinate space, could become a geodesic curve in that space with specified metric coefficients. Further the constraint given by Eq. (19) also puts an upper limit c on the speed computed from any geodesic trace in the distance-time coordinate space or manifold.

B. Physical content of the coordinate planes

A coordinate plane XY may be regarded as physically occupied when the points in the plane represent positions of various material particles or intensities of various interaction fields. A coordinate plane may be regarded as physically empty when none of its points represent the position of any material particle or intensity of any interaction field. A physically empty coordinate plane is an abstract mathematical construct, whereas a physically occupied coordinate plane may be regarded as a physical entity. Let us consider a 2D (thin) metal sheet located in the XY plane of a rectangular Cartesian coordinate system XYT. Let the time axis extend from zero to infinity and let t_p depict the present time on the time axis. Obviously, the t_p marker is continuously moving away from the origin of the time axis. The time zone $t<t_p$ represent the past and the time zone $t>t_p$ represent the future. Now let us take a mental snapshot of the whole range of time axis. We find that the body of the metal sheet is physically located at $t= t_p$ and is not located anywhere in the past or the future time zones. That is, the XY plane representing a section of the XYT manifold at $t=t_p$ can be said to be physically occupied with the thin metal sheet and all other XY planes representing sections of the XYT manifold at $t<>t_p$ are physically empty. This is the standard presentist view of the XYT manifold as per which only the present ($t=t_p$) section of the manifold represent the physical entities and not the whole manifold. As per this viewpoint, the physical state of the thin metal sheet at the next future instant ($t=t_p+\delta t$), evolves from its present ($t=t_p$) state through the operation of physical laws of nature, through the operation of cause and effect.

On the contrary, as per the eternalist view of the XYT manifold, all XY sections of the manifold are supposed to be physically occupied with thin metal sheets. This eternalist viewpoint represents a situation wherein the physical state of all matter particles and their interaction fields, is *predetermined* at all future locations of the metal sheet, or at all XY plane sections for $t>t_p$ of the XYT manifold. This predetermined physical state at all future locations of the metal sheet does not permit a causal evolution of the physical state with progression in time. Further, the notion of predetermined physical state of all matter particles and their interaction fields violates the fundamental principle of cause and effect which is the basis of all scientific study of the universe. Thus the eternalist viewpoint, depicting whole XYT manifold as a physical entity, is fallacious on the grounds of causality violation in the predetermined physical state of the thin metal sheet at all future instants of time.

Suppose, we wish to study the motion of free electrons constrained on the surface of this thin sheet and want to obtain detailed representation for their trajectories or traces of their paths over a finite period of time. For this purpose, we may find it convenient to use 3D XYT manifold to represent the curved traces of the particles under study. While the particles under study are constrained to move in the 2D plane of the metal sheet, their instant to instant position traces can be represented only in the 3D XYT manifold. Study of the geometry of such curved traces can provide us valuable information on the velocities and accelerations of the corresponding particles. However, the geometry of the XYT manifold cannot influence the free electrons constrained on the plane surface of the metal sheet but may influence the *representation* of their dynamic trajectories. The physical phenomenon is occurring only in the metal sheet constrained in the XY coordinate plane located at $t=t_p$. There is no physical phenomenon in the past or the future time zones of the XYT manifold. Hence, logically the past and future time zones of the XYT manifold cannot be regarded as physical entities.

C. Geometrical representation in XYZ-T spacetime manifold

Let us now extend the analogy of 2D plane metal sheet discussed above, to the 3D physical space associated with our solar system. Suppose we wish to study the motion of particles contained within this space and want to obtain detailed representation for their dynamic trajectories or traces over a finite period of time. For this purpose, we may find it convenient to use 4D XYZ-T manifold to represent the instant to instant position traces of the particles under study. While the particles under study are constrained to move in the 3D physical space, their dynamic trajectories can only be represented in the 4D XYZ-T space-time manifold. The geometry of these trajectories can be *correlate***d** with the dynamics of the corresponding particles. However, the geometry of the 4D XYZ-T manifold cannot *influence* the dynamics of particles contained in the 3D physical space but may influence the *representation* of their dynamic trajectories. The physical phenomenon is occurring only in the solar system constrained in the XYZ spatial section at $t=t_p$ of the XYZ-T spacetime manifold. There is no physical phenomenon in the past or the future time zones of the XYZ-T spacetime manifold.

D. Physical content of the spatial sections of space-time manifold

A coordinate space XYZ may be regarded as physically occupied when the points in the space represent positions of various material particles or interaction fields. A coordinate space may be regarded as physically empty when none of its points represent the position of any material particle or interaction field. A physically empty coordinate space is an abstract mathematical construct, whereas a physically occupied coordinate space may be regarded as a physical entity. Let us consider the physical space of our solar system located in a particular spatial section of a Cartesian space-time manifold XYZ-T. Let t_p depict the present instant on the time axis. Obviously, the t_p marker is continuously moving away from the origin of the time axis.[8] The time zone $t<t_p$ represent the past and the time zone $t>t_p$ represent the future.

Taking a mental snapshot of the whole range of time axis, we find that our solar system is physically located at $t=t_p$ and is not located anywhere in the past or future time zones. That is, the 3D XYZ coordinate space representing a spatial section of the 4D XYZ-T manifold at $t=t_p$ can be said to be physically occupied with the solar system and all other 3D XYZ spatial sections of the 4D XYZ-T manifold at $t<>t_p$ are physically empty. This is the standard presentist view of the XYZ-T space-time manifold as per which only the present ($t=t_p$) section of the manifold represent the physical entities and not the whole manifold. As per this viewpoint, the physical state of the solar system at the next future instant ($t=t_p+\delta t$), evolves from its present ($t=t_p$) state through the operation of physical laws of nature, through the operation of cause and effect.

On the contrary, as per the eternalist view of the spacetime, all 3D XYZ sections of the manifold are supposed to be physically occupied with our solar system. This eternalist viewpoint represents a situation wherein the physical state of all matter particles and their interaction fields, is *predetermined* at all future locations of the solar system, or at all 3D XYZ spatial sections for $t>t_p$ of the 4D XYZ-T spacetime manifold. This predetermined physical state at all future locations of the solar system does not permit a causal evolution of the physical state with progression in time. Further, the notion of predetermined physical state of all matter particles and their interaction fields violates the fundamental principle of cause and effect which is the basis of all scientific study of the universe. Thus the eternalist viewpoint, depicting whole 4D XYZ-T spacetime manifold as a physical entity, is fallacious on the grounds of causality violation in the predetermined physical state of the solar system at all future instants of time. Hence, the spacetime continuum is not a physical entity but just an abstract mathematical notion which can neither influence any physical phenomenon nor can its geometry be influenced by any physical phenomenon.

V. CONTINUITY OF DISPLACEMENTS IN A DEFORMABLE SPACE CONTINUUM

Let us, for a moment, accept the eternalist viewpoint and assume the 4D XYZ-T spacetime manifold to be a physical entity. This assumption can then be refuted by showing that a non-zero Riemann 'curvature' tensor in GR always leads to incompatible deformations and discontinuities in the space continuum. Whenever the Riemann tensor of a 4D spacetime manifold is non-zero, the metric coefficients of the space coordinates can no longer remain Euclidean. That is because the distance and time scales get interlinked due to the invariance constraint of Eq. (19). Consider, for example, a Minkowski spacetime manifold with Cartesian coordinates where all metric coefficients are unity and all spatial sections are Euclidean. If a particular solution of EFE yields $g_{tt}<>1$ with $g_{xx}=g_{yy}=g_{zz}=1$, then that solution will no longer satisfy the constraint of Eq. (19) as it will represent the speed of light propagation to be different from c in the Euclidean space. Hence in a 4D spacetime manifold, all metric solutions of EFE, with non-zero Riemann tensor, must also satisfy the constraint of Eq. (19). This constraint will yield non-Euclidean spatial metric coefficients, implying thereby that all 3D spatial sections of the 'curved' spacetime are deformable space continuum. In such a spatial section of the spacetime, let us consider a spherical polar coordinate system with origin at point O and the coordinate parameters r, θ and ϕ. The metric coefficients for this coordinate system in the un-deformed or "gravitation-free" space continuum are given as,

$$g_{rr} = 1\,,\quad g_{\theta\theta} = r^2\,,\quad g_{\phi\phi} = r^2.\mathrm{Sin}^2(\theta). \tag{20}$$

The arc element or the separation distance ds between two neighboring space points P and Q (Fig. 3) in this region will be given by,

$$\begin{aligned}(ds)^2 &= g_{rr}(dr)^2 + g_{\theta\theta}(d\theta)^2 + g_{\phi\phi}(d\phi)^2 \\ &= 1(dr)^2 + r^2(d\theta)^2 + r^2\mathrm{Sin}^2(\theta)(d\phi)^2 \end{aligned} \tag{21}$$

Now, let us assume that a spherically symmetric body of mass M and radius r_0, is located at the origin O of this coordinate system. Due to the gravitational field in its vicinity (i.e. $r > r_0 > 0$), the modified metric coefficients h_{ij} are given by the Schwarzschild solution of EFE as

$$h_{rr} = \frac{1}{1-\frac{2GM}{c^2 r}}\,,\ h_{\theta\theta} = r^2,\ h_{\phi\phi} = r^2\mathrm{Sin}^2(\theta). \tag{22}$$

Thus, the modified radial metric coefficient h_{rr} at any particular space point P(r, θ, ϕ) can be taken as a function of M and its value in the region under consideration is always greater than unity for M>0. The arc element or the modified separation distance ds' between two neighboring space point positions P' and Q' in this region will be given by

$$\begin{aligned}(ds')^2 &= h_{rr}(dr)^2 + h_{\theta\theta}(d\theta)^2 + h_{\phi\phi}(d\phi)^2 \\ &= \frac{(dr)^2}{1-\frac{2GM}{c^2 r}} + r^2(d\theta)^2 + r^2\mathrm{Sin}^2(\theta)(d\phi)^2 . \end{aligned} \tag{23}$$

Therefore, using Eq. (18) we can compute the induced strain tensor components e_{ij} from the modified metric coefficients h_{ij} as

$$2\, e_{rr} = h_{rr} - g_{rr} = (1/(1 - 2GM/c^2r)) - 1 \tag{24}$$

with the factor $2GM/c^2r <<1$; Eq. (24) will get simplified to

$$e_{rr} = GM/c^2r,\ \ e_{\theta\theta} = h_{\theta\theta} - g_{\theta\theta} = 0,\quad e_{\phi\phi} = h_{\phi\phi} - g_{\phi\phi} = 0, \tag{25}$$

and $$e_{\theta\phi} = e_{\phi\theta} = e_{r\phi} = e_{\phi r} = e_{r\theta} = e_{\theta r} = 0 \tag{26}$$

This set of strain tensor components constitutes the strain field induced in the region of space continuum where the gravitational field of M has modified the metric coefficients to h_{ij}.

A. Incompatibility of the induced strain components

For a complete description of the strained state of the space continuum, we must be able to uniquely determine the displacement vector field **U** from the specified strain tensor components. For this the strain tensor components e_{ij}, are required to satisfy a set of integrability or compatibility conditions. The displacement vector components obtained from the integration of strain components, must be single valued, finite and continuous functions of coordinates and must satisfy physical constraints over the boundary of the region of space under consideration. It can be easily seen that the radial strain components e_{rr} given by Eq. (25), with all other components being zero, cannot satisfy the required compatibility conditions. In order to highlight this problem, let us consider the relative displacement vector **U** that gives rise to the strain components e_{rr}, $e_{\theta\theta}$ and $e_{\phi\phi}$. If u^r is the only non-zero component of the displacement vector **U**, then the strain components dependent on u^r are given by,

$$e_{rr} = \partial u^r/\partial r\ ; \quad e_{\theta\theta} = u^r/r \quad \text{and} \quad e_{\phi\phi} = u^r/r\ . \tag{27}$$

Obviously, if the radial strain component e_{rr} is non-zero, the radial displacement component u^r must be non-zero. But once the radial displacement component u^r is non-zero, the tangential strain components $e_{\theta\theta}$ and $e_{\phi\phi}$ cannot be zero. This precisely is the incompatibility of the strain components e_{rr}, $e_{\theta\theta}$ and $e_{\phi\phi}$ induced by the static gravitational field of a spherically symmetric body of mass M. Therefore, the specification of metric coefficients (22) as per the Schwarzschild solution is *physically* invalid and unacceptable on the grounds of incompatible induced strain components.

Let us now examine whether this incompatibility is limited to the strain components induced by the Schwarzschild metric or is applicable to all strain components induced by pseudo-Riemannian metric obtained from EFE. As noted above, all 3D spatial sections of the curved spacetime are also curved in the sense that the Riemann-Christoffel tensor R_{ijkl} composed from the spatial metric coefficients (h_{ij}) will also be non-zero. As shown in Eq. (18) above, the strain components e_{ij} are given by the difference between the Riemannian metric coefficients h_{ij} and the Euclidean metric coefficients g_{ij} of the undeformed continuum as

$$e_{ij} = \tfrac{1}{2}\{h_{ij} - g_{ij}\}. \tag{28}$$

We need to examine the compatibility of the metric induced strain components e_{ij}. According to Saint Venant's integrability or compatibility conditions for a continuous media, the infinitesimal or linear strain tensor components e_{ij} must satisfy following differential equations.[3]

$$e_{ij,kl} + e_{kl,ij} - e_{ik,jl} - e_{jl,ik} = 0 \qquad (\text{with } i,j,k,l \rightarrow 1 \text{ to } 3), \tag{29}$$

If we compose Christoffel 3-index symbols [ij,k] of the first kind and Γ^k_{ij} of the second kind from the symmetric strain tensor components e_{ij}, then Eq. (29) can be written as

$$\partial_l[ki, j] - \partial_i[kl, j] = 0\ . \tag{30}$$

The Riemann-Christoffel tensor R_{jkli} composed from strain components e_{ij} and expressed in terms of Christoffel 3-index symbols, is given by

$$R_{jkli} = \partial_l[ki, j] - \partial_i[kl, j] + \Gamma^{\alpha}_{kl}[ji, \alpha] - \Gamma^{\alpha}_{ki}[jl, \alpha]. \tag{31}$$

Since Saint Venant's compatibility conditions (30) were developed for infinitesimal or linear strain components, the products of strain components and their derivatives could be neglected. However, in the general case of finite or nonlinear strain, the integrability or compatibility conditions are extended from Eq. (30) to include the product terms as

$$\partial_l[ki, j] - \partial_i[kl, j] + \Gamma^{\alpha}_{kl}[ji, \alpha] - \Gamma^{\alpha}_{ki}[jl, \alpha] = 0$$

Or, $$R_{jkli} = 0 \tag{32}$$

Thus to meet the standard compatibility conditions on finite strain components e_{ij}, the Riemann tensor composed from e_{ij} must be a zero tensor.[9] This can be true only if both metrics of Eq. (28), namely, g_{ij} and h_{ij}, are Euclidean, which however contradicts the basic postulate of curved spacetime in GR. Hence, all strain components in the space continuum, induced by the pseudo-Riemannian metric, will fail to satisfy the integrability or compatibility conditions, leading to discontinuities in the induced displacements.[10] Therefore, *if we assume* the 4D spacetime manifold to be a physical entity, we end up with physically invalid discontinuities in the space continuum. Hence the 4D spacetime manifold cannot be a physical entity.

VI. SPACETIME MANIFOLD AS A GRAPHICAL TEMPLATE

A. Differential scaling of coordinate axes

As already discussed in section II above, the shape of an exponential curve on a uniform scale graph can be changed to a straight line on a logarithmic scale graph. Due to the nonlinear or differential scaling of the coordinate axes, a function of the form $y=a.x^b$ will *appear* as a straight line on a log-log graph. Let us plot the trajectory of an object, falling vertically on a gravitating body of mass M, as a Y-T graph such that the Y-axis represents height and the T-axis represents time. We find this trajectory to be a parabolic curve. Now taking a cue from the log-log graph, we can choose a suitable differential scale along Y and T axes such that the parabolic trajectory changes into a straight line on the differential scale graph. This shows that the *shape of free-fall trajectory* of an object moving in a gravitational field, can be changed or adjusted through suitable adjustment of the differential scale along the coordinate axes.

However, such a change in shape of free-fall trajectory does not remain unique unless the differential scales along different coordinate axes are interlinked through some unique constraint. One such constraint imposed on the differential scales or the corresponding metric coefficients along different coordinate axes, is given at Eq. (19). If the differential scales or the corresponding metric coefficients are constrained through Eq. (19), then the shape of free-fall trajectory of an object moving in a gravitational field, can be changed to become a geodesic through suitable adjustment of the metric coefficients. We may use such a differential scale graph as a template for obtaining the geodesic trajectory of any other object falling vertically on a gravitating body of mass M.

Let us now consider the trajectory of an object, falling in a Cartesian 2D X-Y plane on a gravitating body of mass M. The plot of this trajectory on a 3D XY-T linear scale manifold, will again be a parabolic space curve. We can choose a suitable differential scale or the metric coefficients along the X,Y and T axes, in conjunction with the invariance constraint of Eq. (19), such that the parabolic space trajectory on linear scale manifold, changes into a geodesic on the differential scale manifold. Let us make a template of this differential scale manifold. The differential scale or the magnitude of the unit vectors along any particular axis of an orthogonal coordinate system is given by the square-root of the corresponding metric coefficient for that coordinate axis. For obtaining the trajectory of any other object falling in X-Y plane on a gravitating body of mass M, we just need to set the initial position and velocity of this object on the XY-T differential scale template manifold and compute the geodesic curve of the required trajectory. However, for computing trajectories of objects moving in the gravitational field of a body of different mass M', the differential scaling factor or the metric coefficients of the template manifold must be adjusted accordingly to account for different acceleration profile.

B. Differential manifolds to obtain geodesic trajectories

We can extend this methodology for obtaining trajectories of particles moving in 3D physical space, under the gravitational field of a gravitating body of mass M. For this we can first obtain a differential scale 4D manifold XYZ-T as a template by correlating its metric coefficients with the mass M, in conjunction with the invariance constraint of Eq. (19), such that the Newtonian trajectories in the given gravitational field appear as geodesic curves in this template manifold. Now, to obtain the trajectory of any other object in the given gravitational field, we can set the initial position and velocity of the object in the template manifold and then compute the trajectory as a geodesic curve through that position. Of course, we need to adjust the differential scale or the metric coefficients of this template manifold according to the mass M of the gravitating body to account for different acceleration profiles. This is precisely what has been attempted through Einstein Field Equations (EFE) in the spacetime model of GR. Further, to ensure a constant speed of light propagation in all coordinates, a pseudo-Riemannian 4D spacetime manifold, with an invariance constraint of Eq. (19), has been used in GR. This feature may be regarded as an improvement over the Newtonian gravitation, whereby the speed of propagation of gravitational influence is limited to the speed of light.

In GR, the pseudo-Riemannian 4D spacetime manifold is used as an abstract mathematical differential scale template manifold for getting the trajectories of particles as geodesic curves. The differential scale or metric coefficients of this 4D template manifold are correlated through EFE with the mass-energy density in the physical space, to simulate the particle trajectories with geodesic curves in a gravitational field. It may be emphasized here that the correlation between the mass-energy density and the metric coefficients of the 4D template manifold as established through EFE, is essentially an empirical correlation. That is, the EFE do not represent the correlation between the mass-energy density and the metric coefficients of the 4D template manifold that could be deduced from the application of any established law of physics. The validity of such correlation between the mass-energy density and the metric coefficients of the 4D template manifold as established through EFE, can only be demonstrated through accurate simulation of particle trajectories with geodesic curves in a Newtonian gravitational field. Hence, only those solutions of the EFE can be regarded as of any practical significance which can accurately simulate the particle trajectories with geodesic curves in a Newtonian gravitational field. All other solutions of EFE may be regarded as speculative.

The notion of spacetime curvature corresponds to the mathematical situation wherein Riemann tensor R^{i}_{jkl} computed from the metric coefficients of the spacetime template manifold, is non-zero. The metric coefficients corresponding to the non-zero Riemann tensor R^{i}_{jkl}, depict the differential or non-linear scaling along the coordinate axes, to ensure the geometrical representation of an acceleration profile as a geodesic curve. In GR the 4D spacetime differential manifold has been *projected* as the "real world" or a "physical reality" in which its metric is governed by the mass-energy content in space and the consequent geodesics in spacetime guide the motion of material particles in the physical space. Since in Newtonian gravitation the motion of material particles is governed by the local gravitational potential, the metric coefficients in spacetime manifold are effectively treated as gravitational field components, that govern the shape of geodesic curves. As such, the metric coefficients in spacetime manifold are also *assumed* to be physical entities in GR, which are *mystically* governed by EFE. This projection of the 4D spacetime differential manifold as the real world constitutes the most crucial step in *presenting* the abstract spacetime model as a theory of gravitation in GR.

Hence, the general depiction of the non-zero value of Riemann tensor R^{i}_{jkl} as "spacetime curvature" for enunciating a theory of gravitation, gives rise to following misleading connotations about 4D spacetime manifold.

a) The 4D spacetime template manifold is *assumed* to be a physical entity, wherein the universe embedded in 3D physical space is supposed to exist at all points of the "time" axis. In this "block view" of spacetime, the metric induced deformation of the spacetime is *perceived* as spacetime curvature. However, this block view of spacetime violates the principle of causality and hence is invalid. Therefore, the 4D spacetime manifold of GR cannot be regarded as a physical entity.

b) With non-zero Riemann tensor of the 4D spacetime manifold, the metric coefficients of the space coordinates can no longer remain Euclidean. The popular notion of "curvature" of space implies the physical deformation of the space continuum. However, such Riemannian metric induced physical deformation of the space continuum, leads to discontinuities and voids in the continuum which are physically not valid.

Hence, the 4D spacetime manifold cannot be a physical entity.

c) The variable metric of the time coordinate is *assumed* to be a physical effect, depicted through a physical influence of gravitational field on natural cyclic processes used for measurement of time. However, this violates the fundamental notion of time, as a *relative* measure of change.

It may therefore, be concluded that the 4D spacetime model of GR has been used as an abstract 4D template manifold to obtain trajectories of particles as geodesic curves. The mystic connotations associated with this spacetime model may be attributed to the fallacious notion that depicts spacetime as a physical entity, a physical 4D continuum in which the universe, embedded in 3D physical space, is *assumed* to exist at all points on the time axis. As discussed above, the 4D spacetime manifold cannot be regarded as a physical entity on the grounds of causality violation and the curvature induced discontinuities in the space continuum. However, the incorporation of an upper speed limit c, in the spacetime model of GR, may be regarded as an advancement over the Newtonian model of gravitation.

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