

Elongation of Moving Bodies

V.N. Strel'tsov strlve@sunhe.jinr.ru
Laboratory of High Energies
Joint Institute for Nuclear Research
Moscow Region 141980, Dubna
RUSSIA

Abstract: It is marked that the special relativity theory correlates a four-component quantity to a material rod. The corresponding limiting transition from Minkowski's 4-geometry to Euclid's 3-geometry (justified in the rest frame) is provided by vanishing the time component. It is emphasized that the interval (pseudo-length) as a Lorentzian scalar must not depend on velocity. In particular, the space-like interval is equal to the rod length at rest. In a moving frame, its space part (the rod length in motion) because of the negative sign (pseudo-Euclideanness) is always greater than the interval itself. And this means that bodies elongate (but do not contract) in motion.

Keywords: special relativity theory, rod length, 4-dimensional geometry.

“A material rod is physically not a spatial thing but a space-time configuration.”

M. Born [1]

“The three-dimensional geometry becomes a chapter of four-dimensional physics.”

H. Minkowski [2]

The aim of the present article is to give an exact (accurate) representation of the part of modern special relativity theory (SRT) that touches upon the behavior of the space sizes of moving bodies. To the readers showing interest in this theory from the generally scientific viewpoint but being unable to use the mathematical apparatus of 4-dimensional geometry.

4-vector rod. The sense of the afore quoted passages lies factually in that a time component should be added, according to SRT, to three space components (coordinates) describing the rod. Moreover, this addition must be without violations of the theory statements, in particular, the key demand of the Lorentz covariance. The indicated demand is automatically fulfilled if the formed four-component quantity represents a (space-like) 4-vector. Otherwise, the rod length in SRT is defined by the space part of the 4-vector

$$l_i = (ct, x, 0, 0) \quad (1).$$

For simplicity, the rod is oriented and moves along the X-axis, therefore $y=z=0$ and x is the rod length (l). On the other hand, it is evident that the Minkowski 4-geometry must transform to the Euclidean 3-geometry at small velocities, or in the limit $v \rightarrow 0$, i.e. when transiting to the rest frame (S^*). This means in the mathematical language that space-like 4-vectors must transform to the corresponding 3-vectors, i.e., the condition $t^* = 0$ must be fulfilled. As a result, based on eq.(1), for a resting rod we have

$$l_i^* = (0, l^*, 0, 0) \quad (2).$$

The introduction of Minkowski's geometry in RT means that the former "pre-relativistic" invariant -the length (distance) - loses this property. As a result, it cannot any longer be used as the unique characteristic for the classification of rods since, for example, the equality of lengths does not mean at all the identity of rods. (As they may be in different states of

motion.) The interval takes this function.

Interval (pseudolength) of a rod. Thus, one should speak more correctly about the interval of a rod in SRT instead of the rod length. The interval is the main invariant of SRT, and so it is also named the fundamental invariant. The metric of Minkowski's space is said to be defined by the interval squared:

$$s^2 = x^2 - c^2 t^2 \quad (3).$$

By definition, the invariant is the quantity which does not change when transiting from one inertial reference frame to another one. (Therefore, the interval makes it possible to classify rods independently of their state of motion.) As this transition is related to changing the motion velocity, interval invariance must mean its independence of velocity, i.e., constancy (see, e.g., [3,4]). Since, according to SRT, the length of a moving rod depends on velocity a constant (independent of velocity) can only define the rod interval. Such is solely the length of the immovable rod. Therefore,

$$s = l^* \quad (4),$$

i.e., the immovable rod measures a space-like interval, and the immovable clock a time-like interval [5].

The elongation formula. Based on eqs. (3) and (4), we conclude that the space projection (the rod length in motion) is always greater than the interval itself (the rod length at rest) because of the negative sign in the expression for interval (its pseudo-Euclideaness).

Incidentally, already leaning upon eq. (2) and the Lorentz transformation,

$$x = (x^* + vt^*)\gamma \quad (5),$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$, for the length of a moving rod we obtain

$$l = l^* \gamma \quad (6),$$

whence it follows that bodies elongate (but do not contract) while in motion.

Note that at one time formula (6) was obtained on the basis of the concept of covariant (radar) length (see, e.g., [6,7]) leaning upon the "radar definition" of the moving rod length, which is different from the traditional one.

Conclusion. The covariant definition of the moving rod length leads to the elongation equation for longitudinal sizes of moving bodies. Whereas the traditional definition (leading to the contraction formula) contradicts the interval invariance [4] and Piccard-Kessler's experiment [8].

References:

1. Born M. - **Einstein's Theory of Relativity**. Dover, N.Y., 1962, p.253.
2. Minkowski H. - Z.Phys., 1909, v.10, p.104.
3. Strel'tsov V.N., Khvastunov M.S. - Izv. vuzov. Fizika, 1995, v.38, p.125.
4. Strel'tsov V.N. - **Einstein's Definition of Length Contradicts Interval Invariance**. JINR E2-96-72, Dubna, 1996; Apeiron, 1998, v.5, p.209.
5. Mandel'shtam L.I. - **Lectures on Optics, Theory of Relativity and Quantum Mechanics**. Nauka, M., 1972, p.293.
6. Strel'tsov V.N. - Found.Phys., 1976, v.6, p.293.
7. Idem - Sov.J.Part.Nucl., 1991, v.22, p.522.
8. [Idem - J. of Theoretics](#), 2002, v.4, #5.

[Journal Home Page](#)