An Alternative Electrodynamics to the Theory Special Relativity

Musa D. Abdullahi
Umaru Musa Yar'adua University, P.M.B. 2218, Katsina, Nigeria
e-mail: musadab@outlook.com

For an electron of mass m and charge $-e$ moving at time t with velocity $\mathbf{v}$ and acceleration $d\mathbf{v}/dt$ in an electric field of magnitude E, the accelerating force is proposed, in accordance with Newton's second law of motion, as $\mathbf{F} = eE(\mathbf{c} - \mathbf{v})/c = m(d\mathbf{v}/dt)$. The vector $\mathbf{c}$ is the velocity of light and $(\mathbf{c} - \mathbf{v})$ is the relative velocity of the electrostatic force with respect to the moving electron. The electron may move in a straight line to reach the limiting speed c with $\mathbf{F}$ reducing to 0, or it can revolve in a circle at constant speed. The relativistic mass-velocity formula is shown to be correct for circular revolution only and that the "*mass*" in that formula is not a physical quantity but the ratio of electrostatic force $(-eE)$ to centripetal acceleration $(-v^2/r)$ in a circle of radius r. This ratio can become infinitely large for motion in a circle of infinite radius, which is a straight line. An alternative electrodynamics is developed for an electron accelerated to the speed of light at constant mass and with emission of radiation. Radiation occurs if there is a change in the energy of an electron and, as such, circular revolution of an electron, round a central force of attraction, is made stable without recourse to quantum mechanics.

Keywords: Aberration of electric field, acceleration, electric charge, force, mass, radiation, velocity.

1. Introduction

There are now three systems of electrodynamics. Classical electrodynamics is applicable to electrically charged particles moving at a speed which is much slower than that of light. Relativistic electrodynamics is for particles moving at a speed comparable to that of light. Quantum electrodynamics is for atomic particles moving at very high speeds. There should be one consistent system of electrodynamics applicable to all particles moving up to the speed of light.

Classical electrodynamics is based on the second law of motion, originated by Galileo Galilei in 1638 [1], but enunciated by Isaac Newton [2]. The theory of special relativity was formulated mainly by Albert Einstein [3, 4] and the quantum theory was initiated by Max Planck [5]. Relativistic electrodynamics reduces to classical electrodynamics at low speeds but the relativity and quantum theories are incompatible at high speeds. Both the relativity and quantum theories, therefore, cannot be correct. One of the theories or both theories may be wrong. Indeed, special relativity is under attack by physicists: Beckmann [6] and Renshaw [7]. This paper introduces an alternative electrodynamics, applicable to a charged particle, moving in an electric field, at speeds up to that of light, with mass of a particle remaining constant.

According to Newton's second law of motion, a particle can be accelerated by a force to a speed greater than that of light with its mass remaining constant. But experiments with electron accelerators have shown that no particle, not even the electron, the lightest particle known in nature, can be accelerated beyond the speed of light. The theory of special relativity explains this limitation by positing that the mass of a particle increases with its speed, becoming infinitely large at the speed of light. That since an infinite mass cannot be accelerated any faster by any force, the speed of light is the limit to which a body can be accelerated.

The relativistic mass-velocity formula is:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_o \tag{1}$$

where m is the mass of a particle moving with speed v, m_o is the rest mass, c is the speed of light in a vacuum and γ is the Lorentz factor. Equation (1), where m is a physical quantity, becoming infinitely large at the speed of light, is the bone of contention in this paper. The difficulty with infinite mass, at the speed of light, $v = c$, in equation (1), is the Achilles' heel of the theory of special relativity. Resolving this difficulty, by allowing a moving particle to reach the speed of light with its mass remaining constant, is the main aim and purpose of this paper.

The proponents of special relativity just ignore the problem with equation (1). They say that it is the momentum, not the mass, which increases with speed. They avoid the difficulty altogether by arguing that the speed never really reaches that of light c, or that particles moving at the speed of light (photons) have zero rest mass. But electrons are easily accelerated and have been accelerated to practically the speed of light as demonstrated in 1964 by William Bertozzi [8] using a linear accelerator of 30 MeV energy. Electron accelerators, betatrons and electron synchrotrons of over 100 BeV, have been built and operated with electrons moving at the speed of light for all practical purposes.

A most remarkable demonstration of the existence of a universal limiting speed, equal to the speed of light c, was in an experiment by William Bertozzi of the Massachusetts Institute of Technology. The experiment showed that electrons accelerated through energies of 15 MeV or over, attain, practically, the speed of light. Bertozzi measured the heat energy J developed when a stream of accelerated electrons hit an aluminium target at the end of their flight path, in a linear accelerator. He found the heat energy released to be nearly equal to the potential energy P lost, to give $P = J = K$. Bertozzi identified J as solely due to the kinetic energy K lost by the electrons, on the assumption that the accelerating force on an electron of charge $-e$ moving in an electric field of magnitude E, is $-eE$, independent of its speed.

Bertozzi might have made a mistake in equating the heat energy J with the kinetic energy K of the electrons. The energy equation should have been $P = J = K + R$, where R was the energy radiated. Radiation, propagated at the speed of light, also caused heating effect upon impinging at the same point or on the same target as the accelerated electrons. This radiation is a result of aberration of electric field.

1.1. Aberration of Electric Field

Fig. 1 depicts an electron of charge $-e$ and mass m, moving at a point P with velocity $\boldsymbol{v}$, in an electrostatic field of intensity $\boldsymbol{E}$ due to a stationary source charge $+Q$ at an origin O. For motion at an angle θ to the accelerating force $\boldsymbol{F}$, the electron is subjected to aberration of electric field. This is a phenomenon similar to aberration of light discovered by the English astronomer James Bradley in 1728 [9]. In aberration of electric field, as in aberration of light, the direction of the electric field, indicated by the velocity vector $\boldsymbol{c}$, appears shifted by an aberration angle a, from the instantaneous line $\boldsymbol{PO}$, such that:

$$\sin\alpha = \frac{v}{c}\sin\theta \tag{2}$$

where the speeds v and c are the magnitudes of the velocities $\boldsymbol{v}$ and $\boldsymbol{c}$ respectively. The reference direction is the direction of the accelerating force $\boldsymbol{F}$ along the line **PO**. Equation (2) was first derived by James Bradley. Aberration of electric field, which is missing in classical and relativistic electrodynamics, is used in the formulation of the alternative electrodynamics.

The result of aberration of electric field is that the accelerating force on a moving electron depends on the velocity of the electron. If the accelerating force reduces to zero at the speed of light, that speed becomes the ultimate limit, in accordance with Newton's first law of motion. Also, the difference between the accelerating force and the electrostatic force $-eE$ (on a stationary electron) gives the radiation reaction force, from which the radiation power is derived, in contrast to Larmor formula of classical electrodynamics.

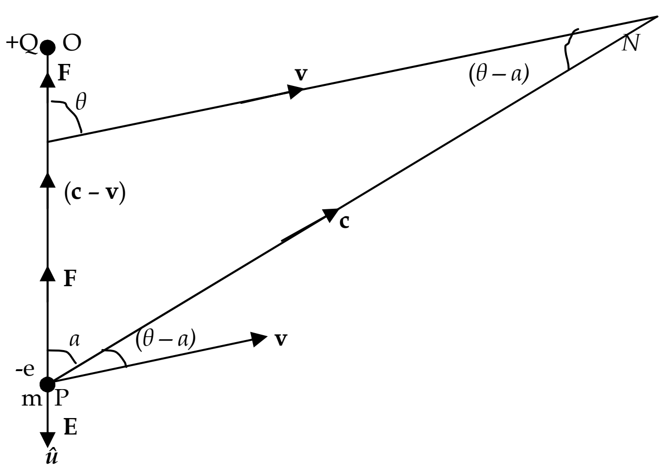

Fig.1. Vector diagram depicts angle of aberration a as a result of an electron of charge $-e$ and mass m moving at a point P with velocity $\boldsymbol{v}$ at an angle θ to the accelerating force $\boldsymbol{F}$. The unit vector $\hat{\boldsymbol{u}}$ is in the direction of the electrostatic field of intensity $\boldsymbol{E}$ due to a charge Q fixed at an origin O.

1.2. Larmor Formula

Larmor formula of classical electrodynamics, described by Griffith [10], gives the radiation power R_p of an accelerated electron as proportional to the square of its acceleration. For an electron revolving with constant speed v in a circle of radius r, with centripetal acceleration of magnitude v^2/r, Larmor classical formula gives $R_p = (e^2/6\pi\varepsilon_o r^2)v^4/c^3$, where ε_o is the permittivity of vacuum. Special relativity adopted this formula [10] and gives radiation power $R = \gamma^4 R_p$, where the Lorentz factor γ is defined in equation (1). The factor γ^4 means that the radiation power increases explosively as the speed v approaches that of light c.

According to Larmor formula, the hydrogen atom, consisting of an electron revolving round a positively charged nucleus, would radiate energy as it accelerates and spirals inward to collide with the nucleus, leading to the collapse of the atom. But atoms are the most stable entities known in nature. Use of Larmor formula was unfortunate as it led physics astray early in the 20th century. It required the brilliant hypotheses of Niels Bohr's [11] quantum mechanics to stabilize the Rutherford's [12] nuclear model of the hydrogen atom.

In the alternative electrodynamics, there is no need for Bohr's quantum theory to stabilize the nuclear model of the hydrogen atom. In this paper it is shown that circular revolution of an electron, with constant speed, round a positively charged nucleus is inherently stable and without irradiation. Radiation comes only if there is change in potential energy of a charged particle moving in an electric field.

2. Equations of Motion in Classical Electrodynamics

The accelerating force $\boldsymbol{F}$ exerted on an electron of charge $-e$ and mass m moving at time t with velocity $\boldsymbol{v}$ and acceleration $(d\boldsymbol{v}/dt)$, in an electrostatic field of intensity $\boldsymbol{E}$, in accordance with Newton's second law of motion. is:

$$\mathbf{F} = -e\mathbf{E} = m\frac{d\mathbf{v}}{dt} \tag{3}$$

For an electron accelerated in a uniform electrostatic field of constant intensity $\boldsymbol{E}$ in the direction of unit vector $\hat{\boldsymbol{u}}$, equation (3) becomes:

$$\mathbf{F} = -eE\hat{\mathbf{u}} = -m\frac{dv}{dt}\hat{\mathbf{u}} \tag{4}$$

Equation (4) is a first order differential equation with solution:

$$v = at \tag{5}$$

where speed $v = 0$ at time $t = 0$ and $a = eE/m$ is a constant.

An electron moving with velocity $\boldsymbol{v}$ in the direction of the field, suffers a deceleration and the equation of motion becomes:

$$\mathbf{F} = -eE\hat{\mathbf{u}} = m\frac{dv}{dt}\hat{\mathbf{u}} \tag{6}$$

The solution of equation (6) for an electron decelerated from speed of light c by a uniform field E:

$$v = c - at \tag{7}$$

The electron is decelerated to a stop in time $t = c/a$.

3. Equations of Motion in Relativistic Electrodynamics

The accelerating force $\boldsymbol{F}$ exerted on an electron of charge $-e$ and mass m, moving with velocity $\boldsymbol{v} = v\hat{\boldsymbol{u}}$ at time t in an electrostatic field of intensity $\boldsymbol{E}$, is:

$$\mathbf{F} = -eE\hat{\mathbf{u}} = -\frac{d}{dt}(mv)\hat{\mathbf{u}} \tag{8}$$

where mass m increases with speed v, so that:

$$\mathbf{F} = -eE\hat{\mathbf{u}} = -\frac{d}{dt}\frac{m_o v}{\sqrt{1-\frac{v^2}{c^2}}}\hat{\mathbf{u}} \tag{9}$$

For a constant field E, equation (9) is also a first order differential equation with solution as:

$$v = \frac{at}{\sqrt{1+\frac{a^2t^2}{c^2}}} \tag{10}$$

where speed $v = 0$ at time $t = 0$ and a = eE/m is the acceleration constant. Equation (10) makes the speed of light c the ultimate limit as time $t \to \infty$.

In relativistic electrodynamics, an electron moving at the speed of light c cannot be decelerated and stopped by any force. Such a moving electron continues to move at the speed of light, gaining potential energy without losing kinetic energy.

4. Equations of Motion in the Alternative Electrodynamics

The force exerted on an electron, moving with velocity $\mathbf{v}$, by an electrostatic field, is propagated at the velocity of light $\mathbf{c}$ relative to the source charge and transmitted with velocity ($\mathbf{c} - \mathbf{v}$) relative to the electron. The electron can be accelerated to the velocity of light $\mathbf{c}$ and no faster. In Fig. 1 the electron may be accelerated in the direction of the force with $\theta = 0$ or it may be decelerated against the force with $\theta = \pi$ radians or it can revolve in a circle, perpendicular to the accelerating force, where $\theta = \pi/2$ radians.

The accelerating force $\mathbf{F}$ (see Figure 1), on an electron of charge $-e$ and mass m moving at time t with velocity $\mathbf{v}$ and acceleration $(d\mathbf{v}/dt)$, in an electrostatic field of magnitude E and intensity $\mathbf{E} = E\hat{\mathbf{u}}$, in the direction of unit vector $\hat{\mathbf{u}}$, is proposed as given by the vector equation and Newton's second law of motion, thus:

$$\mathbf{F} = \frac{eE}{c}(\mathbf{c}-\mathbf{v}) = m\frac{d\mathbf{v}}{dt} \tag{11}$$

where $\mathbf{c}$ is the velocity of light at aberration angle α to the accelerating force $\mathbf{F}$ and $(\mathbf{c} - \mathbf{v})$ is the relative velocity of transmission of the force with respect to the moving electron. The force propagated at velocity of light $\mathbf{c}$, cannot "catch up" and "impact" on an electron also moving with velocity $\mathbf{v} = \mathbf{c}$. With no force on the electron, it continues to move with constant speed c, in accordance with Newton's first law of motion. Equation (11) may be regarded as an extension, amendment or modification of Coulomb's law of electrostatic force between two electric charges, taking into consideration their relative velocity.

Equation (2) links the angle θ with the aberration angle α (Fig.1). Equation (11) is the basic expression of the alternative electrodynamics. Expanding this equation, by taking the *modulus* of the vector $(\mathbf{c} - \mathbf{v})$, with respect to the angles θ and α, gives:

$$\begin{aligned}\mathbf{F} &= \frac{eE}{c}(\mathbf{c}-\mathbf{v}) \\ &= \frac{-eE}{c}\sqrt{c^2+v^2-2cv\{\cos(\theta-\alpha)\}}\hat{\mathbf{u}} = m\frac{d\mathbf{v}}{dt}\end{aligned} \tag{12}$$

where $(\theta - \alpha)$ is the angle between the vectors $\mathbf{c}$ and $\mathbf{v}$.

4.1. Equations of Rectilinear Motion

For an accelerated electron where $\theta = 0$, equations (2) and (12) give the force $\mathbf{F}$ as:

$$\mathbf{F} = -eE\left(1-\frac{v}{c}\right)\hat{\mathbf{u}} = -m\frac{dv}{dt}\hat{\mathbf{u}} \tag{13}$$

This is a first order differential equation. The solution of equation (13) for an electron accelerated by a uniform electric field of constant magnitude E, from initial speed u, is:

$$v = c - \{c-u\}\left\{\exp\left(-\frac{at}{c}\right)\right\} \tag{14}$$

For acceleration from zero initial speed equation (14) becomes:

$$\frac{v}{c} = 1 - \exp\left(-\frac{at}{c}\right) \tag{15}$$

where $a = eE/m$ is a constant. Figure 2.C1 is a graph of v/c against at/c for equation (14). The electron will be accelerated to an ultimate speed equal to that of light c.

The distance $x = \int v(dt)$, covered in time t by an electron accelerated from zero initial speed, is obtained by integrating equation (15), thus:

$$x = ct + \frac{c^2}{a}\left\{\exp\left(-\frac{at}{c}\right)-1\right\} \tag{16}$$

For a decelerated electron where $\theta = \pi$ radians, equations (2) and (12) give the decelerating force $\mathbf{F}$ as:

$$\mathbf{F} = -eE\left(1+\frac{v}{c}\right)\hat{\mathbf{u}} = m\frac{dv}{dt}\hat{\mathbf{u}} \tag{17}$$

Solving the differential equation (17) for an electron decelerated from speed u, by a uniform electric field, gives:

$$v = \{c+u\}\left\{\exp\left(-\frac{at}{c}\right)\right\} - c \tag{18}$$

For an electron decelerated from speed of light c, equation (18) becomes:

$$\frac{v}{c} = 2\exp\left(-\frac{at}{c}\right) - 1 \tag{19}$$

Figure 2.C2 is a plot of v/c against at/c according to equation (19). The electron will be decelerated to a stop (v = 0) in time $t = \frac{c}{a}\ln 2 = \frac{0.693c}{a}$, having lost kinetic energy $0.5mc^2$, equal to the potential energy gained plus the energy radiated.

Figure 2 shows a graph of v/c against at/c for an electron accelerated from zero initial speed, or an electron decelerated from speed of light c, by a uniform electric field: the solid lines, (A1) and (A2) according to classical electrodynamics, the dashed curve (B1) and line (B2) according to relativistic electrodynamics and the dotted curves (C1) and (C2) according to equations (15) and (19) of the alternative electrodynamics.

The distance $x = \int v(dt)$, covered in time t by an electron decelerated from the speed of light c, is obtained by integrating equation (8) to give:

$$x = \frac{2c^2}{a}\left\{1-\exp\left(-\frac{at}{c}\right)\right\} - ct \tag{20}$$

In equations (19) and (20) it is seen that an electron entering a uniform decelerating field at a point with speed c, will come to a stop in time $t = 0.693c/a$, at a distance $X = 0.307c^2/a$ from the point of entry, having lost kinetic energy equal to $0.5mc^2$, gained potential energy equal to $eEX = 0.307mc^2$ and radiated energy equal to $0.193mc^2$. The electron will come back to the starting point (x = 0) in time $t = 1.594c/a$, with speed $0.594c$ and kinetic energy $0.176mc^2$, having lost potential energy equal to $eEX = 0.307mc^2$ and radiated energy equal to $0.131mc^2$. The electron will then be accelerated to the speed of light c, as the ultimate limit, with emission of radiation. These results are not obtainable from the point of view of classical or relativistic electrodynamics

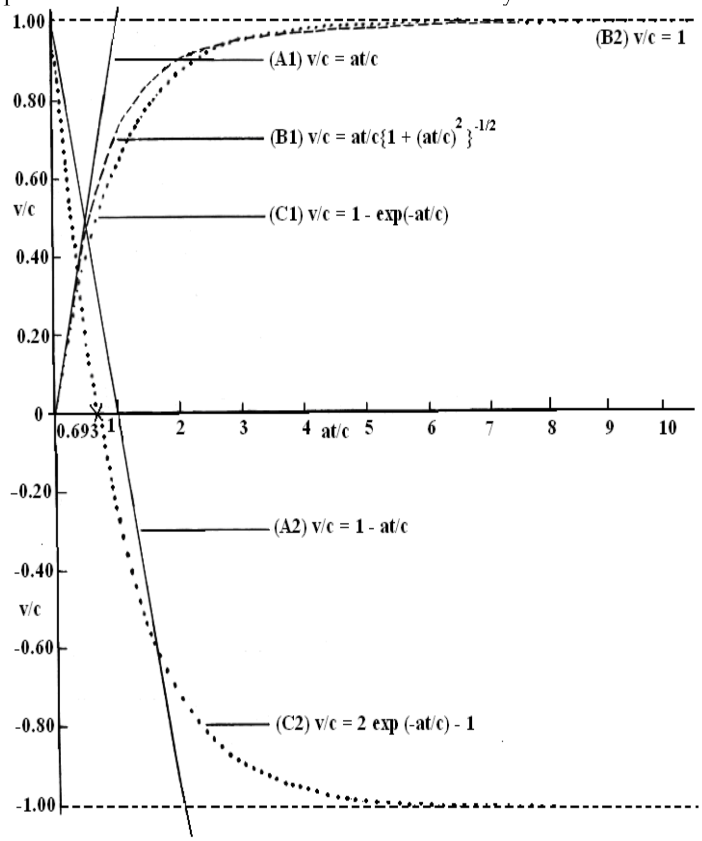

Fig. 2 *v/c (speed in units of c)* against *at/c (time in units of c/a)* for an electron of charge $-e$ and mass $m = m_o$ accelerated from zero initial speed or decelerated from the speed of light c, by a uniform electrostatic field of magnitude E, where $a = eE/m$; the lines (A1) and (A2) according to classical electrodynamics (equations 5 and 7), the dashed curve (B1) according to relativistic electrodynamics (equation 10), dashed line (B2) for relativistic electrodynamics and the dotted curves (C1) and (C2) according to equations 15 and 19 of the alternative electrodynamics presented here.

4.2. Equations of Circular Motion

For $\theta = \pi/2$ radians, equation (12) gives revolution in a circle of radius r with constant speed v and acceleration $(-v^2/r)\hat{\mathbf{u}}$. Equations (2) and (12), with mass $m = m_o$ (rest mass) and noting that $\cos(\pi/2 - a) = \sin a = v/c$, give the accelerating force $\mathbf{F}$ as:

$$\mathbf{F} = -eE\sqrt{1-\frac{v^2}{c^2}}\hat{\mathbf{u}} = -m\frac{v^2}{r}\hat{\mathbf{u}} = -m_o\frac{v^2}{r}\hat{\mathbf{u}}$$

$$eE = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v^2}{r} = \varsigma\frac{v^2}{r} \tag{21}$$

$$\varsigma = \frac{eEr}{v^2} = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} \tag{22}$$

Equation (1) for 'm' and equation (22) for ζ (zeta) are identical but obtained from two different points of view. In equation (1), for relativistic electrodynamics, the quantity 'm' increases with speed v, becoming infinitely large at speed c. In equation (22), for the alternative electrodynamics, mass m remains constant at the rest mass m_o, and the quantity $\zeta = \{(eE)/(v^2/r)\}$ is the ratio of magnitude of the radial electrostatic force $(-eE)$ on a stationary electron, to the centripetal acceleration $(-v^2/r)$ in circular motion. This quantity ζ may become infinitely large at the speed of light c, without any difficulty. At the speed of light, that is $(v = c)$, the electron moves in a circle of infinite radius, a straight line, to make 'm' or ζ also infinite without any problem or difficulty.

In classical electrodynamics, radius r of circular revolution for an electron of charge $-e$ and mass m, in a radial electric field of magnitude E due to a positively charged nucleus, is:

$$r = \frac{mv^2}{eE} = \frac{m_o v^2}{eE} = r_o \tag{23}$$

where $m = m_o$, the rest mass, is a constant and r_o is the classical radius of revolution. In relativistic electrodynamics, where mass m is supposed to vary with speed v in accordance with equation (1), the radius of revolution becomes:

$$r = \frac{mv^2}{eE} = \frac{m_o v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o \tag{24}$$

In the alternative electrodynamics, where $m = m_o$ is a constant, the radius r of revolution, obtained from equation (24), becomes:

$$r = \frac{m_o v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \frac{m\ v^2}{eE\sqrt{1-\frac{v^2}{c^2}}} = \gamma r_o \tag{25}$$

Relativistic electrodynamics and the alternative electrodynamics give the same expression for radius of revolution in circular motion as $r = \gamma r_o$ but for different reasons.

5. Radiation Reaction Force and Radiation Power

The difference between the accelerating force $\mathbf{F}$ on a moving electron and the electrostatic force $-e\mathbf{E}$ on a stationary electron, is the radiation reaction force $\mathbf{R}_f = \mathbf{F} - (-e\mathbf{E})$, that is always present when a charged particle is accelerated by an electric field. This is analogous to a frictional force, which always opposes motion. A simple and useful expression for radiation reaction force $\mathbf{R}_f$ is missing in classical and relativistic electrodynamics and it makes all the difference. The radiation force, $-\mathbf{R}_f$, gives the direction of emitted radiation from an accelerated charged particle. For rectilinear motion, with $\theta = 0$ (Fig.1), equation (11) gives $\mathbf{R}_f$, in the direction of unit vector $\hat{\mathbf{u}}$, as:

$$\mathbf{R}_f = -\frac{eE}{c}(c-v)\hat{\mathbf{u}} + eE\hat{\mathbf{u}} = \frac{eEv}{c}\hat{\mathbf{u}} = -\frac{eE}{c}\mathbf{v} \tag{26}$$

In rectilinear motion, with $\theta = \pi$ radians, $\mathbf{R}_f = -(eEv/c)\hat{\mathbf{u}} = -(eE\mathbf{v}/c)$, same as equation (26).

Radiation power is $R_p = -\mathbf{v}.\mathbf{R}_f$, the scalar product of $\mathbf{R}_f$ and velocity $\mathbf{v}$. The scalar product is obtained, with reference to Fig. 1, as:

$$R_p = -\mathbf{v}\cdot\mathbf{R}_f = -\mathbf{v}\cdot\left\{\frac{eE}{c}(\mathbf{c}-\mathbf{v})+e\mathbf{E}\right\}$$

$$R_p = eEv\left\{\cos\theta - \cos(\theta-\alpha)+\frac{v}{c}\right\} \tag{27}$$

For rectilinear motion with $\theta = 0$ or $\theta = \pi$ radians, equations (2) for θ and a and equation (27) give radiation power as:

$$R_p = -\mathbf{v}.\mathbf{R}_f = eE\frac{v^2}{c} \tag{28}$$

Positive radiation power, as given by equation (28), means that energy is radiated in accelerated and decelerated motions.

In circular revolution, where the velocity $\boldsymbol{v}$ is orthogonal to $\boldsymbol{E}$ and $\boldsymbol{R_f}$, the radiation power R_p (scalar product of $\boldsymbol{v}$ and $\boldsymbol{R_f}$) is zero, as can be ascertained from equations (2) and (27) with $\theta = \pi/2$ radians and $\cos(\theta - a) = \sin a = v/c$. Equation (27) is significant in the alternative electrodynamics. It makes circular revolution of an electron, round a central force of attraction, as in Rutherford's nuclear model of the hydrogen atom, without radiation and stable, without recourse to Bohr's quantum theory.

Equations (26), (27) and (28) are the radiation formulas of the alternative electrodynamics. These equations are in contrast to those of classical electrodynamics where the radiation force is proportional to the rate of change of acceleration (*Abraham-Lorentz formula*) and the radiation power is proportional to the square of acceleration (*Larmor formula*). There is no formula for radiation reaction force in relativistic electrodynamics but special relativity adopted a modified Larmor formula. Let us now consider potential energy and radiation for an accelerated or decelerated electron in three systems of electrodynamics

6. Potential Energy and Radiation in Classical Electrodynamics

The accelerating force on an electron of charge $-e$ and constant mass m, moving at time t with speed v and acceleration dv/dt, in the opposite direction of an electrostatic field of magnitude E, is given, in accordance with Coulomb's law of electrostatic force and Newton's second law of motion, by equation (4):

$$eE = m\frac{dv}{dt}$$

For rectilinear motion of an electron in the direction of a displacement x, we obtain the differential equation:

$$eE = m\frac{dv}{dt} = mv\frac{dv}{dx} \tag{29}$$

The potential energy P lost by the moving electron or work done on the electron, in being accelerated with constant mass m, through a distance x from an origin $(x = 0)$, to a speed v from rest, is given by the definite integral:

$$P = \int_0^x eE(dx) = m\int_0^v v(dv) \tag{30}$$

Integrating, equation (30) gives:

$$\int_0^x eE(dx) = P = \frac{1}{2}mv^2$$

This is equal to the kinetic energy of the electron.

$$\frac{P}{mc^2} = \frac{1}{2}\left(\frac{v}{c}\right)^2 \tag{31}$$

Here, with no consideration of energy radiation, the potential energy P lost is equal to the kinetic energy gained by an accelerated electron.

In classical electrodynamics, an electron, moving at the speed of light c, can be decelerated to a stop and may be accelerated in the opposite direction to reach a speed greater than $-c$. The potential energy P gained in decelerating an electron from the speed of light c to a speed v, within a distance x in a field E, is:

$$-\int_0^x eE(dx) = -P = m\int_c^v v(dv) = \frac{1}{2}m(v^2 - c^2)$$

$$\frac{P}{mc^2} = \frac{1}{2}\left(1-\frac{v^2}{c^2}\right) \tag{32}$$

Neither is energy radiation considered here.

7. Potential Energy and Radiation in Relativistic Electrodynamics

The kinetic energy K gained by an electron or the work done, in being accelerated by an electric field E, through a distance x, to a speed v from rest, is the potential energy P lost. There is no consideration of energy radiation. The kinetic energy K of a particle of mass m and rest mass m_o moving with speed v is given by the relativistic equation:

$$\int_0^x eE(dx) = K = P = mc^2 - m_oc^2$$

$$\int_0^x eE(dx) = P = \frac{m_oc^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_oc^2$$

$$\frac{P}{m_oc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \tag{33}$$

where m_o is the rest mass (at $v = 0$) and c the speed of light in a vacuum. The increase in kinetic energy is supposed to be accounted for by the increase in mass. Bertozzi's experiment was conducted to verify the relativistic equation (33) and it did so in a remarkable way.

In relativistic electrodynamics, an electron moving at the speed of light c (with infinite mass), cannot be stopped by any decelerating force. The electron continues to move at the same speed of light c, gaining potential energy without losing kinetic energy, contrary to the principle of conservation of energy.

8. Potential Energy and Radiation in the Alternative Electrodynamics

In an alternative electrodynamics, the accelerating force $\boldsymbol{F}$, with reference to Fig. 1, is given by equations (2) and (12): For acceleration in a straight line, these equations with $\theta = 0$, give:

$$\mathbf{F} = -eE\left(1-\frac{v}{c}\right)\hat{\mathbf{u}} = -m\frac{dv}{dt}\hat{\mathbf{u}} \tag{34}$$

The scalar equation is:

$$eE\left(1-\frac{v}{c}\right) = m\frac{dv}{dt} = mv\frac{dv}{dx} \tag{35}$$

The potential energy P lost in accelerating the electron, through a distance x, to a speed v from rest, is given by the integral:

$$P = \int_0^x eE(dx) = \int_0^v mv \frac{dv}{1 - \frac{v}{c}} \tag{36}$$

Resolving the right-hand integral into partial fractions, we obtain:

$$P = mc \int_0^v \left(\frac{1}{1 - \frac{v}{c}} - 1 \right) dv \tag{37}$$

$$P = -mc^2 \ln\left(1 - \frac{v}{c}\right) - mcv \tag{38}$$

$$\frac{p}{mc^2} = -\ln\left(1 - \frac{v}{c}\right) - \frac{v}{c} \tag{39}$$

Energy radiated R is obtained by subtracting the kinetic energy $K = ½ mv^2$ from P, thus:

$$R = P - K = -mc^2 \ln\left(1 - \frac{v}{c}\right) - mcv - \frac{1}{2}mv^2$$

$$R = -mc^2 \left\{ \ln\left(1 - \frac{v}{v}\right) + \frac{v}{c} + \frac{v^2}{2c^2} \right\} \tag{40}$$

Equation (39), for the alternative electrodynamics, should be compared with equation (33) for relativistic electrodynamics and equation (32) for classical electrodynamics.

For a decelerated electron, equations (2) and (12), with $\theta = \pi$ radians, give:

$$\mathbf{F} = -eE\left(1 + \frac{v}{c}\right)\hat{\mathbf{u}} = m\frac{dv}{dt}\hat{\mathbf{u}} \tag{41}$$

$$eE\left(1 + \frac{v}{c}\right) = -m\frac{dv}{dt} = -mv\frac{dv}{dx} \tag{42}$$

Potential energy P gained in decelerating the electron through distance x, from speed c to v, is:

$$P = -\int_0^x eE(dx) = \int_c^v -mv \frac{dv}{1 + \frac{v}{c}} \tag{43}$$

Resolving the integrand into partial fractions and integrating, gives:

$$P = -mc \int_c^v \left(1 - \frac{1}{1 + \frac{v}{c}} \right) dv \tag{44}$$

$$P = mc^2 \ln\frac{1}{2}\left(1 + \frac{v}{c}\right) + mc^2\left(1 - \frac{v}{c}\right) \tag{45}$$

$$\frac{P}{mc^2} = \ln\frac{1}{2}\left(1 + \frac{v}{c}\right) + \left(1 - \frac{v}{c}\right) \tag{46}$$

Energy radiated is the difference between kinetic energy lost and potential energy gained, thus:

$$R = \frac{1}{2}m(c^2 - v^2) - mc^2 \ln\frac{1}{2}\left(1 + \frac{v}{c}\right) - mc^2\left(1 - \frac{v}{c}\right)$$

$$R = -mc^2 \left\{ \frac{1}{2} + \frac{v^2}{2c^2} + \ln\frac{1}{2}\left(1 + \frac{v}{c}\right) - \frac{v}{c} \right\} \tag{47}$$

In classical and relativistic electrodynamics, there is no consideration of energy radiation for an electron decelerated from the speed of light.

9. Speed Versus Potential Energy in Bertozzi's Experiment

In the experiment performed by William Bertozzi [7], the speed v of high-energy electrons was determined by measuring the time T required for them to traverse a distance of 8.4 metres after having been accelerated through a potential energy P inside a linear accelerator. Bertozzi's experimental data, reproduced below, demonstrated that electrons accelerated through potential energies over 15 MeV attain, practically, the limiting speed of light c.

RESULTS OF BERTOZZI'S EXPERIMENTS WITH ELECTRONS ACCELERATED THROUGH ENERGY P IN A LINEAR ACCELERATOR
(m_oc^2 = 0.5 MeV, v = 8.4/T m/sec)

P (MeV)	P/m_oc^2	T x 10^{-8} sec.	v x 10^8 m/sec	v/c
0.5	1	3.23	2.60	0.87
1.0	2	308	2.73	0.91
1.5	3	2.92	2.88	0.96
4.5	9	2.84	2.96	0.99
15.0	30	2.80	3.00	1.00

10. Speed Versus Potential Energy in Three Systems of Electrodynamics

A graph of P/mc^2 (*potential energy in units of* mc^2) against v/c (*speed in units* c), is shown in Fig. 3; the solid curves (A1 and A2) in accordance with classical electrodynamics (equations 31 and 32), the dashed curve (B1) according to relativistic electrodynamics (equation 33) and the dotted curves (C1) and (C2) according to the alternative electrodynamics (equations 39 and 46). The three solid circles are the results of Bertozzi's experiment [8].

11. Concluding Remarks

Relativistic electrodynamics and the alternative electrodynamics appear to be in agreement for an accelerated electron in rectilinear motion (equations 10 and 15), as depicted in Figs. 2 and 3. Also Bertozzi's experimental results appear to be in agreement with relativistic electrodynamics (equation 33) and the alternative electrodynamics (equation 38) for an electron in accelerated rectilinear motion, as depicted in Fig. 3. The two systems of electrodynamics demonstrate clearly the speed of light c as a limit; relativistic electrodynamics on the basis of mass of a moving particle increasing to become infinitely large at the speed of light and the alternative electrodynamics on the basis of accelerating force reducing to zero at the speed of light.

Relativistic electrodynamics and the alternative electrodynamics give the same expression for radius of circular revolution as γr_o for an electron round a centre of force of attraction, where Lorentz factor γ is defined in equation (1) and r_o is the classical radius as expressed in equation 25. In circular revolution of a charged particle, decrease in accelerating force with speed, in accordance with the alternative electrodynamics, has the same effect (increase in radius) as apparent increase of mass with speed in accordance with special relativity. This may explain the apparent agreement between relativistic electrodynamics and revolution of charged particles (electrons and protons) in cyclic accelerators. At the speed of light the accelerating force on a revolving particle reduces to zero, in accordance with the alternative electrodynamics, and the electron moves in a circle of infinite radius, which is a straight line. This is in contrast to relativistic electrodynamics where increase in radius is misinterpreted as being the result of mass increasing with speed becoming finitely large at the speed of light.

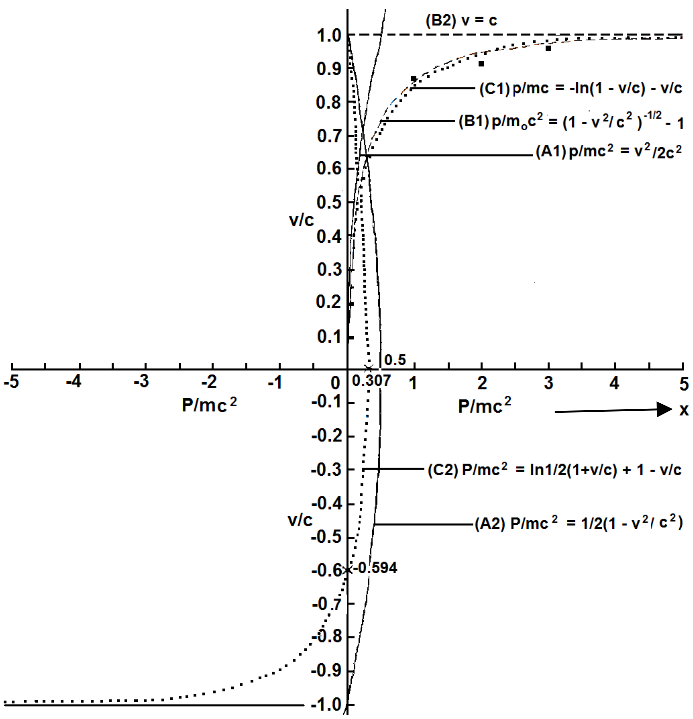

Fig. 3 *v/c* (*speed in units of c*) against *P/mc²* (*potential energy in units of mc²*) for an electron of mass *m* accelerated from zero initial speed or decelerated from the speed of light *c*, the solid curves (A1 and A2) according to classical electrodynamics (equation 31 and 32), the dashed curves (B1) according to relativistic electrodynamics (equation 33) and the dotted curves (C1 and C2) according to the alternative *electrodynamics* (equations 39 and 46). The sold dots are the result of Bertozzi's experiment (Table 1).

In the alternative electrodynamics, the force exerted on an electric charge by an electric field depends on the velocity of the charge. This is tantamount to modifying Coulomb's law of electrostatic force, taking into consideration the relative velocity between two electric charges moving in space. This is not the case in classical and relativistic electrodynamics where the Coulomb force of attraction or repulsion, between two electric charges, is made independent of their velocities.

The question now is: "which one of the electrodynamics is correct?" The answer may be found in the motion of electrons decelerated from the speed of light *c*. According to classical electrodynamics, an electron of mass *m* entering a retarding field at a point (x = 0), with speed *c*, is brought to rest after losing kinetic energy $0.5mc^2$, equal to the potential energy gained, without energy radiation. The electron may then be accelerated backwards to reach the point of entry with speed $-c$ and may reach a speed greater than $-c$ without radiation of energy.

According to relativistic electrodynamics, an electron moving at the speed of light (with infinitely large mass and energy), cannot be stopped by any force. The electron should continue to move at the speed of light gaining potential energy without losing kinetic energy.

In the alternative electrodynamics an electron moving at the speed of light, on entering a retarding field at a point (x = 0), is easily brought to rest after gaining potential energy equal to $0.307mc^2$ and radiating energy equal to $0.193mc^2$ (Fig. 3 equation 45). The electron may then be accelerated backwards to return to the point of entry with speed $-0.594c$, losing potential energy equal to $0.307mc^2$, gaining kinetic energy equal to $0.176mc^2$ and radiating energy equal to $0.131mc^2$ (Fig. 3, equation 46).. The electron may then be accelerated to reach an ultimate speed $-c$ with radiation of energy, and in all cases, its mass remaining constant. An electron moving at the speed of light, in an electric field, acquires the characteristic of light.

For accelerated electrons, special relativity and the alternative electrodynamics appear to be in agreement. The picture is completely different for decelerated electrons. It is energy radiation which makes all the difference. In the alternative electrodynamics, an electron moving at the speed of light is easily brought to rest by a decelerating field. Such an electron being stopped and turned back on its track, by a decelerating field, invalidates special relativity.

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