CHAPTER IV

THE THEORY OF RELATIVITY

1. Introduction.

The Mössbauer effect involves the theory of relativity in several ways. The thermal shift (see Section 2.8) has been variously described in terms of the special theory. The gravitational red shift experiment demonstrates the equivalence of mass and energy and involves the general theory of relativity. The rotor experiment described in this thesis is subject to relativistic time dilatation and is a most accurate test of the fundamental postulates of the special theory. As the thermal shift and the time dilatation observed in the rotor experiment can be considered using arguments arising from both the special and the general theory, they also demonstrate the cohesive overlap of the two theories.

2.a. The Lorentz Transformation.

Before Michelson and Morley conducted the first aether drift experiment in 1887, it had generally been assumed that electromagnetic waves propagated with the speed of light c relative to a fundamental aether considered to be at rest in respect to Newtonian absolute space. This view was further substantiated by early experiments on the aberration of star light (Bradley, 1728; Airy, 1871; Lodge, 1892) which demonstrated that the aether was not carried along by the earth. According to the classical concepts, Maxwell's equations were form invariant under a set of Galilean transformations.

The failure to detect the classical aether did produce a dilemma, as it became impossible to unify Maxwell's equations with Newtonian kinematics. Even though Fitzgerald (1890) and Lorentz (1895) could explain the null result of the experiment by postulating that bodies moving with velocity v, relative to the absolute frame, are contracted by $\left(1-v^2/c^2\right)^{\frac{1}{2}}$ in the direction of motion, the dilemma was not resolved until 1904 when Lorentz showed that under a particular set of transformations, now named after its originator, Maxwell's equations stayed form-invariant. Lorentz did try to interpret the contraction phenomenon physically, but did not realize the underlying significance of the transformations.

It was left to Einstein (1905) to show the fundamental significance of the transformations. Starting from a critical analysis of the measuring process, he was able to derive the transformation properties from the two fundamental postulates:

i. That the laws of physics are the same or invariant in all inertial frames of reference, and

ii. That the speed of light (in vacuum) is a constant independent of the inertial reference system used or the state of motion of the source or the observer.

This approach in conjunction with the work of Poincare and Minkowski provides a most consistent and general framework for relating physical phenomena as observed in different inertial frames of reference. Using the 4-vector notation, the Lorentz transformation from frame X to X^1, the latter moving with velocity v relative to X, is described by

$$x_\mu{}^1 = \sum_{\lambda=1}^{4} a_{\mu\lambda} x_\lambda \qquad \text{... 2.1}$$

where $x = \left(x_1,\ x_2,\ x_3,\ ict\right)$ are the four coordinates of frame X, and similarly, x^1 are the coordinates of X^1. For the case of v being parallel to x_1

$$a_{\mu\lambda} = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\gamma\beta & 0 & 0 & \gamma \end{bmatrix} \qquad \text{... 2.2}$$

with $\beta = v/c$ and $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$

In accordance then with the first postulate, all laws of physics are covariant under a Lorentz transformation. This means that scalar quantities are Lorentz invariant, 4-vectors transform in accordance with Equation 4.2.1 and tensors like

$$F^1_{\mu\mathrm{n}} = \sum_{\lambda_1\sigma=1}^{4} a_{1\mu\lambda} a_{\mathrm{n}\sigma} F_{\lambda\sigma}$$

where in particular $F_{\lambda\sigma}$ is the electromagnetic field strength tensor and gives the transformation properties of the electric and the magnetic field.

The Lorentz transformation has thus become the basis of all of relativistic mechanics, and it would appear desirable to test its general validity as accurately as is possible today. The Michelson-Morley experiment gave the original impetus for the two postulates, and it can also be regarded as the most direct test of the transformation. Other more indirect experiments test the consequences of the Lorentz transformation and have in recent years most strikingly verified the predictions of the special theory of relativity.

2.b. Time Dilatation and the Doppler Effect.

According to the Lorentz transformation, the time $t_\circ$ measured in a moving frame is slowed down by a factor γ in comparison with the time measured in a stationary frame. A number of experiments have been conducted to compare the life time $\tau_\circ$ of unstable particles, pions and muons, moving at relativistic speeds with the life time of these particles at rest (Rossi, et al., 1941; Durbin, et al., 1952; Frisch and Smith, 1963). They have established the time dilatation factor to within 10%. The most convincing experiment is that by Farley, et al., (1967) in which the decay of muons in a storage ring was directly monitored as a function of time. They found that $\tau_\circ/\tau = 25.15 \pm 0.03$ is in satisfactory agreement with $\gamma = 26.72$.

In the above experiments time dilatation is measured directly, whereas in experiments involving the propagation of electromagnetic radiation, the dilatation factor can be deduced from the modified Doppler shift formula. Because of the Lorentz invariance of the phase of an electromagnetic wave, with wave number $\underline{k}$ and frequency n, the relativistic Doppler shift can be obtained directly by transforming the phase $k_\mu x_\mu = \sum_{i=1}^{3} k_i x_i - 2\pi \mathrm{n} t$ from the stationary to the moving frame. An observer moving with velocity $\underline{v}$ relative to the source of radiation will then measure a frequency

$$\mathrm{n}^1 = \mathrm{n}\gamma\left(1 - \frac{\hat{n}\cdot\underline{\mathrm{v}}}{c}\right) \qquad \text{... 2.3}$$

where $\hat{n}$ is the unit vector in the direction of wave propagation. This Doppler shift is in essence the classical Doppler shift modified by the dilatation factor. For $\hat{n}\cdot\underline{\mathrm{n}} = 0$ the time dilatation factor emerges directly. It has been observed by Ives and Stilwell (1938, 1941) when they measured the frequency of the H_β lines emitted by H_2 ions moving at relativistic speeds $(\beta = 0.005)$. An improved version of this experiment (Mandelberg and Witten, 1962) confirmed the expected result to an accuracy of 5%.

A similar effect has been observed using the Mössbauer effect with the source and the absorber mounted on a high-speed rotor. Generalizing Equation 4.2.3 for the case where both the source and the absorber are in motion relative to the laboratory frame, the frequencies n, n^1 of the emitted and the received radiation are related by (Lee and Ma, 1962)

$$\frac{\mathrm{n}^1}{\mathrm{n}} = \frac{\gamma a\left(1-\hat{n}\cdot\underline{U}_a/c\right)}{\gamma_s\left(1-\hat{n}\cdot\underline{U}_s/c\right)} \qquad \frac{\hat{n}\cdot\underline{U}_a/c}{\gamma_s\left(1-\hat{n}\cdot\underline{U}_s/c\right)}$$

$$\approx 1+\frac{(U_a^{\,2}-U_s^{\,2})}{2c^2} \qquad \text{for } \hat{n}\cdot\left(\underline{U}_a-\underline{U}_s\right)=0$$

where the energy shift between source and absorber can be expressed by

$$\frac{\Delta\mathrm{n}}{\mathrm{n}} = \frac{\mathrm{n}_a-\mathrm{n}_s}{\mathrm{n}_s} = \frac{\omega^2}{2c^2}\left(r_s^{\,2}-r_a^{\,2}\right) \qquad \text{... 2.4}$$

$\underline{U}_s$, $\underline{U}_a$ are the velocities of the source and the absorber relative to the laboratory frame of reference. r_s, r_a are their respective radial distances from the axes of rotation, and ω is the angular velocity of the rotor.

The first experiment of this kind was performed by Hay, et al., (1960) using a ^{57}Co (in Fe) source and iron absorber. Later experiments (Hay, 1961; Champeney, et al., 1963) utilized the isomeric shift to verify the sign of the relativistic shift and confirmed its magnitude to within 2%. Kündig (1963) in a similar experiment Doppler shifted the source line to observe the whole transmission spectrum and reached an accuracy of 1.1%.

The thermal shift (see Section 2.8) can also be considered in terms of the transverse Doppler shift. As the characteristic frequencies of the atom in the lattice are 10^{12} sec^{-1}, the terms $\hat{n}\cdot\underline{U}_s$ $\hat{n}\cdot\underline{U}_a$ averaged over the lifetime of the nucleus will be zero so that only the second order shift will remain.

A discussion of time dilatation invariably involves the problem of the clock paradox which arises when considering the reciprocity of time measured in two different inertial frames of reference. The authoritative approach to time dilatation is to regard it as an absolute phenomenon; i.e., not a consequence of the measuring process, as has been suggested by Essen (1957, 1964) and one that can't be traced back to simpler phenomena (Møller, 1952). The problem of reconciling the one-sided time dilatation with the reciprocity of its observation during an out and return journey is then solved by invoking general relativistic arguments (Born, 1924; Tolman, 1934). This view is contested by Dingle (1961, 1967) who maintains that STR, which refutes the existence of an absolute frame of reference, cannot give rise to absolute effects associated with relative motion. It has been suggested (Lord Halsbury, Bondi, 1957) that the clock paradox can be treated without invoking the aspect acceleration by using three inertial frames of reference in motion along a straight line and such an analysis does give the same result as that obtained using general relativity.

Sherwin (1960) maintains that the thermal shift observed with the Mössbauer effect does satisfy the conditions of an out and return journey and so confirms Einstein's predictions. The shift caused by thermal vibrations of the ^{57}Fe nuclei involves non-uniform motion as well as large accelerations of the order of 10^{16}g.

This consistent overlap of the special and the general theory is well brought out by considering the quadratic shift in terms of the scalar gravitational potential at the source and the observer (Pauli, 1958). Two clocks at rest relative to one another but located at different gravitational potentials χ_a, χ_s are related in frequency by (Møller, 1957)

$$\frac{n_a}{n_s} = \left(\frac{1 + 2\chi_a / c^2}{1 + 2\chi_s / c^2} \right)^{\frac{1}{2}} \approx 1 + \frac{\chi_a - \chi_s}{c^2} \qquad \text{... 2.5}$$

$$\frac{\Delta n}{n} = \frac{n_a - n_s}{n_s} = \frac{\chi_a - \chi_s}{c^2}$$

They will differ in frequency by an amount directly proportional to the difference in their respective gravitational potentials.

The scalar gravitational potential at the surface of the earth produces an energy shift of 1.1 x 10^{-16} per meter difference in the vertical height. This gravitational red shift has been observed by Cranshaw, et al., (1960); Pound and Rebka (1960); Cranshaw and Schiffer (1964); and Pound and Snider (1964) with standard deviations of 50%, 10%, 10%, and 1%, respectively.

By employing the equivalence principle, which states that effects arising from gravitational attraction are locally indistinguishable from those arising from acceleration, one can extend the interpretation of χ to include potential differences arising from centifugal forces, in which case one obtains

$$\frac{\Delta n}{n} = \frac{\chi_a - \chi_s}{c^2} = \frac{\omega^2}{2c^2}\left(r_s^{\,2} - r_a^{\,2} \right)$$

a result equivalent to Equation 2.4.

3. The Aether Drift Experiment.

3.1 Historical Considerations.

Einstein developed his theory in the context of an empty space, devoid of any structure and without the need of an absolute frame of reference. Partly as a result of his later work and that of Mach one regards space today as having a distinct structure determined by the distribution and the relative velocities of matter in the universe. That light is affected by this structure has been established by the gravitational red shift experiments; that it propagates in an inertial frame of reference is seen from the Sagnac experiment (Sagnac, 1913). In the latter experiment two light beams were sent in opposite near circular optical paths and then brought to interference. Upon rotating the whole apparatus with angular velocity ω, a fringe shift proportional to ω was observed, from which one can conclude that light does not partake in the motion of the apparatus. Instead, it propagates in an inertial frame of reference in direct analogy with the Foucault pendulum.

These considerations about the nature of light propagation have recently led a number of authors to reconsider the ideas of Lorentz (Builder, 1958; Janossy, 1962, 1964, 1965; Prokhovnik, 1967). Bondi (1962) has pointed out that, cosmologically speaking, there is a preferred frame of reference and that motion relative to it can be readily determined by measuring the Doppler-shifted spectral lines of the stars. The problem of light propagation thus links the special theory of relativity with Cosmology. To understand the nature of light propagation, one must be able to link the two fields by resolving the question of the need for a preferred frame of reference. Does motion relative to the cosmologically preferred frame affect the propagation of light?

This question is unequivocally answered by the special theory, but because the theory does not involve the nature and the effects of the inertial forces experienced by one inertial frame moving relative to another, the question has continued to occupy a number of scientists and still today gives rise to some controversy regarding the interpretation of the special theory of relativity (Prokhovnik, 1967).

Ives (1948) has examined the problem of measuring the velocity of light, in an inertial frame of reference moving with velocity V relative to the cosmological frame. There are in principle two ways of measuring the light velocity. One is to measure it over a return path (Einstein's method, 1905) which has the advantage of using only one timing clock but the disadvantage that it measures only the average velocity. The other method is to conduct a one-way measurement which, as Ives shows, does depend on the method used to synchronize the two timing clocks, but is independent of the velocity V. This is because the rods and clocks, which are in essence used to measure the light velocity, are equally affected by the relativistic factor γ. He was thus able to show the equivalence of the special theory and the Fitzgerald Lorentz contraction theory in a local frame of reference. Ives' work has been extended by Robertson (1949) and Builder (1958). Robertson has shown that Einstein's basic postulates can be deduced from the three basic experiments of (i) Michelson and Morley, (ii) Kennedy and Thorndike (see Section 4.3.3), and (iii) Ives and Stilwell and in fact can be replaced by the assumptions that there exists a fundamental frame of reference and that bodies moving relative to such a frame are contracted by the relativistic factor (Builder, 1958). The phenomenon of time dilatation is then deduced using Einstein's measuring conventions (Builder, 1960) and the relativistic predictions are considered as consequences of the Lorentz transformation. The validity of Einstein's basic postulates can then be inferred from experiments where the one-way velocity of light is measured in an inertial frame as a function of the velocity of the source or the observer, and also as a function of V, the velocity of the inertial frame relative to the cosmological frame.

These considerations led Ruderfer (1960) to propose the present aether drift experiment, as it essentially represented such measurement. It was thought that the experiment might be able to distinguish between the two viewpoints (Møller, 1962) but a more careful analysis (Ruderfer, 1961) showed that both theories give rise to the same prediction of a null result.

It is interesting to note that some of the basic questions regarding the interpretation of the relativistic phenomena still appear not to have been resolved and continue to give rise to controversy. This does demonstrate the need to conduct more accurate experiments that are sensitive to higher-order effects, as is the present experiment.

Prokhovnik (1967) examines the logical consequences of the differing interpretations and comes to the comforting conclusion that it is possible to conceive of a physical model for the universe which does accommodate both viewpoints and where in fact both approaches are valid descriptions of different facets of the universe. The approach of the STR does represent the very fundamentality of the relativistic concept in modern physics, and the neo-Lorentzian viewpoint does shed more light on the cosmological problem.

3.2 Theory of the Aether Drift Experiment.

Following the line of argument of classical physics, one would regard light propagation to be isotropic relative to the fundamental aether, which is assumed to be at rest relative to Newtonian absolute space. In a frame moving with velocity $\underline{V}$ relative to the fundamental frame, the phase velocity $\underline{c}^1{}_p$ and the group velocity of light $\underline{c}^1{}_g$ would then be modified (Møller, 1952, 1957).

$$\underline{c}^1{}_p = c^1{}_p \hat{n}^1 = c\hat{n} - \underline{V}$$
$$\underline{c}^1{}_g = c^1{}_g \hat{e}^1 = c\hat{e} - \underline{V}$$

where $\hat{n}$ and $\hat{n}^1$ are unit vectors in the direction of the wave normal in the two frames; $\hat{e}$ and $\hat{e}^1$ are unit vectors in the direction of propagation of the wave energy similarly in the two frames. In the classical theory $\hat{e}^1 \neq \hat{n}^1$, but because the phase velocity does have the same direction in both reference frames

$$\hat{n}^1 = \hat{n}$$

and in the aether frame is equal to the group velocity

$$\hat{n} = \hat{e}$$

so that

$$c^1{}_p = c - \hat{n} \cdot \underline{V}$$
$$c^1{}_g \hat{e}^1 = c \cdot \hat{n} - \underline{V}$$

and $\hat{n}$ and $\hat{e}^1$ are then related by

$$\hat{n} \approx \hat{e}^1 \left(1 - \frac{\underline{V} \cdot \hat{e}^1}{c} \right) + \frac{\underline{V}}{c} \quad \text{for} \quad \frac{V}{c} << 1 \qquad \text{... 3.1}$$

Assuming then that a light source and an observer are moving with respective velocities $\underline{V}_s$, $\underline{V}_o$ relative to the aether, the observer in his frame will measure the frequency of the light source ν_s as being

$$\nu_\circ = \frac{\nu_s \left(1 - \frac{\hat{n} \cdot \underline{V}_o}{c} \right)}{\left(1 - \frac{\hat{n} \cdot \underline{V}_s}{c} \right)} \qquad \text{... 3.2}$$

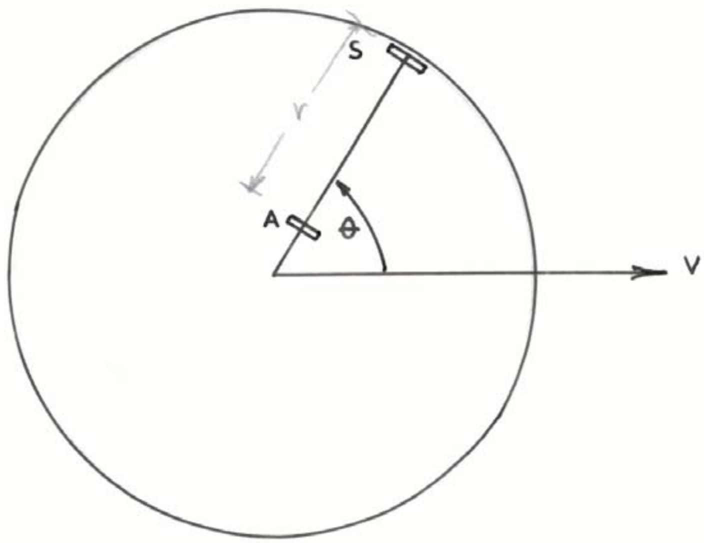

FIGURE 4.1

This Doppler shift, when written in terms of the velocities relative to the laboratory frame, becomes

$$\frac{\Delta\nu}{\nu}=\frac{\nu_{\circ}-\nu_{s}}{\nu_{s}}=\frac{\left(\hat{e}^{1}\cdot\underline{U}\right)}{c}+\frac{\left(\hat{e}^{1}\cdot\underline{U}\right)\left(\hat{e}^{1}\cdot\underline{U}_{s}\right)}{c^{2}}+\frac{\underline{V}\cdot\underline{U}}{c^{2}} \qquad \text{... 3.3}$$

where

$$\underline{U}_{s}=\underline{V}_{s}-\underline{V}$$

$$\underline{U}_{\circ}=\underline{V}_{\circ}-\underline{V}$$

and

$$\underline{U}=\underline{U}_{s}-\underline{U}_{\circ}$$

Equation 4.3.3 has been written in terms of the unit vector $\hat{e}^{1}$, because it is the propagation of the wave energy, which is the observable quantity.

One would ideally then like a geometry that would satisfy

$$\hat{e}^{1}\cdot\underline{U}=0$$

in which case

$$\frac{\Delta\nu}{\nu}=\frac{\underline{V}\cdot\left(\underline{U}_{s}-\underline{U}_{\circ}\right)}{c^{2}} \qquad \text{... 3.4}$$

is the frequency shift one would expect as a first approximation of the classical theory.

In a rotor experiment with a Mössbauer source and absorber mounted rigidly to the rotor as for instance shown in Figure 4.1, the above condition is readily satisfied. In such an experiment the inherent stability of the nuclear transition provides two stable clocks, and the gamma ray a means to compare the rate of both. Hence any effects, arising from the motion of the laboratory frame relative to the cosmological rest frame, producing a local anisotropy in the propagation of light or the nuclear or electromagnet forces acting during the decay of the nuclear state, would become observable.

A way of relating the two clock rates can also be found by considering the phase of the electromagnetic wave as emitted by S and received by A. (Ruderfer, 1960.) In a geometry, as shown in Figure 4.1, the phase can be expressed as

$$\text{j} = \nu_s \left(t - \frac{r}{c + \underline{V} \cdot \hat{n}} \right) \qquad \text{... 3.5}$$

where r is the radial distance between the source and the absorber and is assumed to be constant. The second term in Equation 4.3.5 arises because, in the presence of an aether, the phase velocity of light in the rotor frame will be modulated by V̲.

Taking the time derivative of j to obtain the instantaneous frequency as seen by A, the frequency shift becomes to first order

$$\frac{\Delta \nu}{\nu} = \frac{\underline{V} \cdot \underline{U}}{c_2} \left(1 + \frac{4\underline{V} \cdot \hat{e}^1}{c} + \ . \ . \ . \right) \square \ \frac{\underline{V} \cdot \underline{U}}{c^2} \qquad \text{... 3.6}$$

where $\frac{d\theta}{dt} = \underline{\omega}$, and $\underline{\omega} x \underline{r} = \underline{U} = \underline{U}_s - \underline{U}_a$ with

$$\frac{d\text{n}_s}{dt} = \frac{dr}{dt} = 0$$

This to first order is the same result as obtained before.

In the contraction theory, in which light is similarly thought to propagate relative to some fundamental frame, the frequency shift, expressed by Equation 4.3.6, is canceled by the shift arising from time dilatation. With the source and the observer moving with velocities $\underline{\text{V}}_s$, $\underline{\text{V}}_\circ$ relative to the fundamental frame, the Doppler shift becomes

$$\frac{\Delta \text{n}}{\text{n}} = \frac{U^2{}_s - U_a{}^2}{2c^2} - \frac{\underline{\text{V}} \cdot \underline{U}}{c}$$

i.e., the aether dependent shift is canceled, and only the time dilatation factor remains.

However, for the next higher order term, one has to consider the effect of the contraction of r, which will be of order UV^2/c^3 and would have to be included in the higher order terms of Equation 4.3.6. An experiment sensitive to the higher order terms thus would test the exact cancellation of the time dilatation and the contraction factor making the one-way propagation of light isotropic relative to an inertial frame of reference. Following this line of argument, one again arrives at Einstein's basic postulates.

3.3 Past Experiments.

The first optical experiment by Michelson-Morley (1887) reached an accuracy of 10 km/sec compared to the expected aether drift of 30 km/sec, the velocity of the earth around the sun. The experiment was repeated by Joos (1931) who set a limit on the aether of 1.5 km/sec, and by Kennedy and Thorndike (1932) whose modified interferometer reached a sensitivity of 15 km/sec. The interferometer used by Kennedy and Thorndike was similar to that used by Michelson but had unequal interferometer arms. Miller (1933) contested these results and claimed to have detected an effect of 10 km/sec. This contradictory result was later attributed to systematic errors (Shankland, et al., 1955). More recently modern microwave (Essen, 1955) and laser techniques (Jaseja, et al., 1964) have been used placing a limit of 2.5 km/sec and 1.0 km/sec, respectively. All these experiments are only sensitive to first order terms in v/c.

A very substantial increase in accuracy was achieved when methods became available that are sensitive to the second order term in v/c. The first such experiment was that conducted by Cedarholm, et al. (1958, 1959) who used two ammonia beam masers and set a limit of 30 m/sec. With the advent of the Mössbauer effect and its high inherent stability, a potential increase in sensitivity of four orders of magnitude was envisaged.

Champeney and Moon (1961) and Cranshaw and Hay (1961) set limits of 50 m/sec and 10 m/sec, and later experiments by Champeney, et al. (1963) and Turner and Hill (1964) achieved a sensitivity of 3 m/sec and 16 m/sec, respectively.

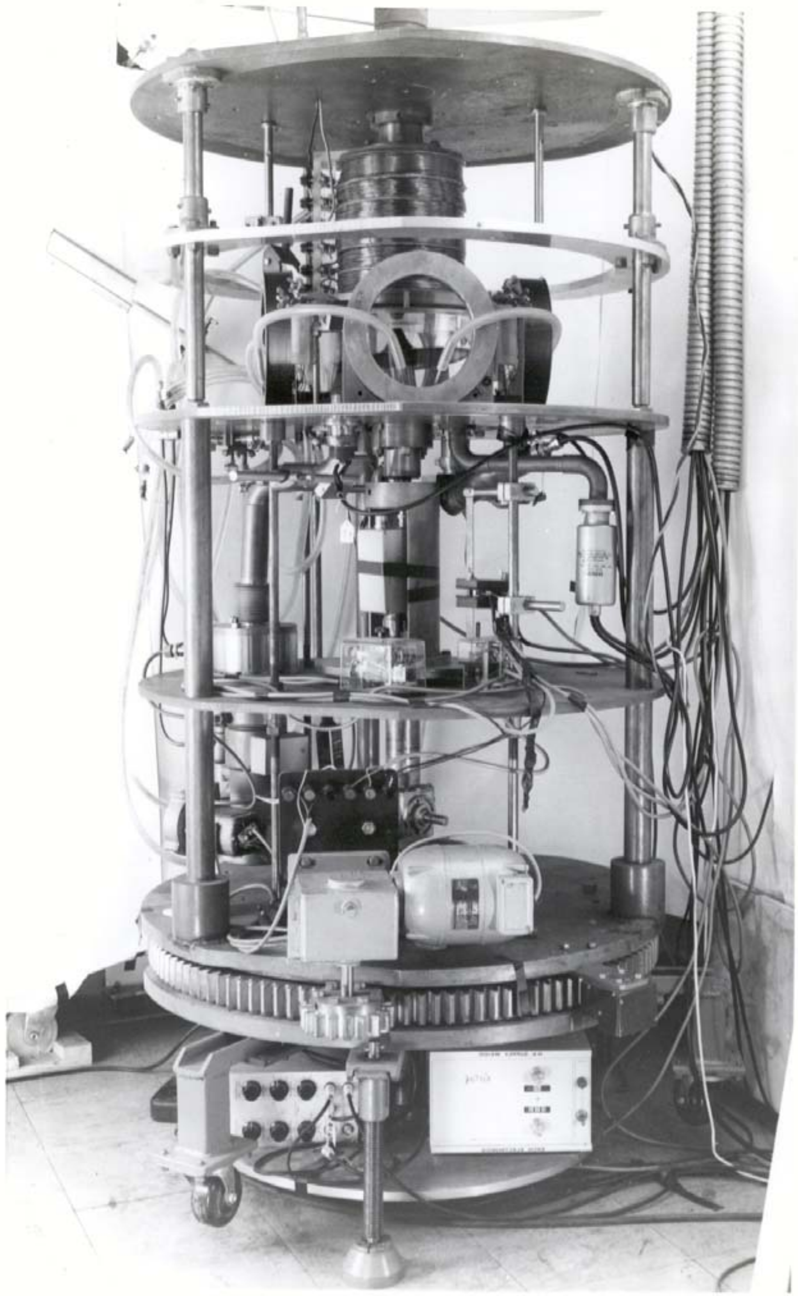

PLATE 5.1